Mathematics

1. R = {(x, y) : x and y were born in Delhi on same day}
R is an equivalence relation.

2. Required no. of subsects
$= {}^{10}C_2 + {}^{10}C_3 = 45 + 120 = 165$

3. $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3} = 0$

4. $(\alpha-\beta)^2 < 5$
$\Rightarrow k^2 - 4 < 5 \Rightarrow k^2 < 9$
$\Rightarrow -3 < k < 3$ ..... (1)
Next, $|k| \geq 2 \Rightarrow k \leq -2$ or $k \geq 2$ ..... (2)
from both, $k \in (-3, -2] \cup [2, 3)$

5. Let α, $k\alpha$ be roots of $x^2 + px + q = 0$
and β, $k\beta$ be roots of $x^2 + \ell x + m = 0$

Clearly, $\frac{\alpha^2}{\beta^2} = \frac{p^2}{\ell^2} = \frac{q}{m} \Rightarrow p^2 m = \ell^2 q$

6. $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n = \omega^n + \omega^{2n} = -1$, n is not multiple of 3

7. Sum = $12(10^0 + 10^1 + 10^2)$
$= 111 \times 12 = 1332$

8. 0.3 + 0.33 + 0.333 + ...
$= \frac{3}{10} + \frac{33}{100} + \frac{333}{1000} + ...$
$= \frac{3}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...\right]$
$= \frac{1}{3}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + \left(1-\frac{1}{1000}\right) + ...\right]$
$= \frac{1}{3}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{\frac{9}{10}}\right] = \frac{1}{3}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

9. $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2) = 0$

10. In an A.P
If $S_m = n$ and $S_n = m$ then $S_{m+n} = -(m+n)$

11. $\frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-(-2i)} = \frac{1+2i}{1+2i} = 1$
Modulus = 1, Argument = 0

12. If graph of quadratic lies entirely above x-axis then D > 0. So both roots are complex.

13. $|z+1| = |z+4-3|$
$\leq |z+4| + |-3|$
maximum value of $|z+1| = 6$

14. $z^2 = 2\bar{z}$
Let $z = x + iy$
Now $x^2 - y^2 + 2xyi = 2x - 2yi$
$\Rightarrow x = -1$ and $y = \pm\sqrt{3}$
So, $z = -1 + \sqrt{3}i$ and $z = -1 - \sqrt{3}i$
are two roots

15. $\cot(\alpha+\beta) = \frac{\cot\alpha - \cot\beta - 1}{\cot\beta + \cot\alpha}$
$= \frac{c-1}{-b} = \frac{1-c}{b}$

16. $-b = \frac{b^2-2c}{c^2}$
$\Rightarrow -bc^2 = b^2 - 2c \Rightarrow 2c = b^2 + bc^2$
$\Rightarrow \frac{2c}{b} = b + c^2 \Rightarrow \frac{2}{b} = \frac{b}{c} + c$
$\Rightarrow c, \frac{1}{b}, \frac{b}{c} \in$ A. P $\Rightarrow \frac{1}{c}, b, \frac{c}{b} \in$ H. P

17. $-\frac{1}{a} = \frac{\frac{1}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} \Rightarrow -\frac{1}{a} = \frac{1-2ca}{c^2} \Rightarrow a - 2ca^2 = -c^2$
$\Rightarrow 2ca^2 = a + c^2 \Rightarrow a, ca^2, c^2 \in$ A. P

18. ${}^8C_1 + {}^8C_2 + ...\ {}^8C_7 = 2^8 - 2 = 254$

19. ${}^3P_2 \times 6! = 6 \times 720 = 4320$

20. $a_n = 2n - 3 \Rightarrow a_5 = 7$

21. 11 + 13 + ... + 99
$= \frac{45}{2}(11 + 99) = 45 \times 55 = 2475$

22. $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$
$= \left(1-\frac{1}{2}\right) + \left(1-\frac{1}{4}\right) + \left(1-\frac{1}{8}\right) + ...$
$= n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$
$= n - \frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{\frac{1}{2}} = n - 1 + 2^{-n}$

23. By property, $(A-B)\cup A = A$, $(A-B)\cap B = \phi$ and $A \subseteq B \Rightarrow A \cup B = B$ are true.

24. $(1p101)^2 + (10q1)_2 = (100\,r00)_2$
Equating, $P = 0$, $q = 1$, $r = 0$

25. $S = \{x : x^2 + 1 = 0, x \in R\}$
$\Rightarrow S = \phi$

26. ${}^nC_4\ x^{n-4}y^4 - {}^nC_5\ x^{n-5}y^5 = 0$

$\Rightarrow {}^nC_4\ x^{n-4}y^4 = {}^nC_5\ x^{n-5}y^5$

$\Rightarrow \frac{x}{y} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$

27. $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} \alpha^2+4 & 4\alpha \\ 4\alpha & \alpha^2+4 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \alpha^3+12\alpha & 6\alpha^2+8 \\ 6\alpha^2+8 & \alpha^3+12\alpha \end{bmatrix}$

$\Rightarrow |A^3| = 125$

$\Rightarrow (\alpha^3+12\alpha)^2 - (6\alpha^2+8)^2 = 125$

putting $\alpha = \pm 3$, $(63)^2 - (62)^2 = 125$ satisfied.

28. $|B^{-1}AB| = |B^{-1}|\ |A|\ |B|$

$= \frac{1}{|B|}\ |A|\ |B| = |A|$

29. Putting $x = 0$

$\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$ is a skew-symmetric matrix of odd order whose determinant value is zero.

30. $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$AA^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = Identity matrix

31. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = 1(9-15) - 2(18-15) + 3(10-5)$

$= -6 - 6 + 15 = 3 \neq 0$
$\Rightarrow$ system has unique solution.

32. $\begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3x+y \\ x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

solving we get, $x = 2$, $y = -2$

$A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

33. $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\begin{vmatrix} 1 & 1 & 1 \\ 0 & xyz & 0 \\ 0 & 0 & xyz \end{vmatrix} = x^2y^2z^2$

34. $x^3 + y^3 = 0 \Rightarrow x^2 - xy + y^2 = 0$

$\Rightarrow \left(\frac{x}{y}\right)^2 - \frac{x}{y} + 1 = 0,\ \frac{x}{y} = -1$

$\Rightarrow \frac{x}{y}$ is one of the cube roots of -1

35. By symmetry B has as many elements as C.

36. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

37. $(1 \times 3)(3 \times 3)(3 \times 1)$
$= (1 \times 1)$

38. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^4 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

39. $\sin A = \frac{3}{5}$

$\Rightarrow \cos A = -\frac{4}{5}$ ($\because$ A lies in 2nd quadrant)

$\therefore \cos^2\frac{A}{2} = \frac{1+\cos A}{2} = \frac{1}{10}$

$\Rightarrow \cos\frac{A}{2} = \frac{1}{\sqrt{10}}$ ($\frac{A}{2}$ lies in 1st Quadrant)

40. $\dfrac{1}{\sin 10°} - \dfrac{\sqrt{3}}{\cos 10°}$

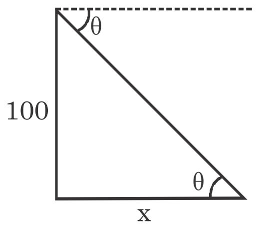
$= \dfrac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10° \cdot \cos 10°}$

$= \dfrac{2 \times 2\left[\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right]}{2\sin 10° \cos 10°}$

$= 4\dfrac{\cos(60° + 18)}{\sin 20°} = 4\dfrac{\cos 70°}{\sin 20°} = 4$

41. $\tan\theta = \dfrac{5}{12}$

$\Rightarrow \dfrac{100}{x} = \dfrac{5}{12} \Rightarrow x = 240$ m



42. $f(x) = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right) = \sqrt{2}\sin\left(x + \frac{\pi}{6} + \frac{\pi}{4}\right)$

f(x) is maximum when $x + \dfrac{\pi}{6} + \dfrac{\pi}{4} = \dfrac{\pi}{2} \Rightarrow x = \dfrac{\pi}{12}$

43. $K = \sin 10° \cdot \sin 50° \cdot \sin 70°$

$= \dfrac{1}{4}\sin 30° = \dfrac{1}{4} \times \dfrac{1}{2} = \dfrac{1}{8}$

44. $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta}$

$= \dfrac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$= \tan\left(\dfrac{\alpha+\beta}{2}\right)$

45. $\dfrac{\sin(\theta + 2\alpha)}{\sin\theta} = \dfrac{1}{3}$

$\Rightarrow \dfrac{\sin(\theta+2\alpha) + \sin\theta}{\sin(\theta+2\alpha) - \sin\theta} = \dfrac{4}{-2} = -2$

$\Rightarrow \dfrac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha} = -2$

$\Rightarrow \dfrac{\tan(\theta+\alpha)}{\tan\alpha} = -2$

$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$

46. $\tan 18° = \dfrac{\sqrt{5} - 1}{\sqrt{10 + 2\sqrt{5}}}$ (Result)

47. $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$\Rightarrow \tan^{-1}\dfrac{2y}{1-y^2} = \tan^{-1}\dfrac{x+z}{1-xz}$

$\Rightarrow \dfrac{2y}{1-y^2} = \dfrac{x+z}{1-xz} = \dfrac{x+z}{1-y^2}$ $(\because y^2 = xz)$

$\Rightarrow 2y = x + z$

$\Rightarrow x, y, z \in$ A.P

But $x, y, z \in$ G.P (given)

$\Rightarrow x = y = z$

48. $\tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$

$= \dfrac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)} = \dfrac{2+1}{1-2\times 1} = \dfrac{3}{-1} = -3$

49. $A + B + C = \pi$

$\sin\left(\dfrac{B+C}{2}\right) = \sin\left(\dfrac{\pi}{2} - \dfrac{A}{2}\right) = \cos\dfrac{A}{2}$ (True)

$\tan\left(\dfrac{B+C}{2}\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{A}{2}\right) = \cot\dfrac{A}{2}$ (True)

Statement 1 and 2 are true.

50. $\sec\theta - \operatorname{cosec}\theta = \dfrac{4}{3}$

$\Rightarrow \dfrac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = \dfrac{4}{3}$

$\Rightarrow \dfrac{(\sin\theta - \cos\theta)^2}{(\sin\theta\cos\theta)^2} = \dfrac{16}{9}$

$\Rightarrow \dfrac{1 - 2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta} = \dfrac{16}{9}$

Let $\sin\theta\cos\theta = x$

$\Rightarrow \dfrac{1-2x}{x^2} = \dfrac{16}{9}$

$\Rightarrow 16x^2 + 18x - 9 = 0$

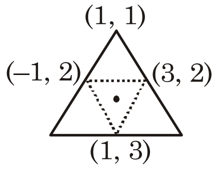
$\Rightarrow (8x - 3)(2x + 3) = 0$

$\Rightarrow x = \dfrac{3}{8}, x = \dfrac{-3}{2}$

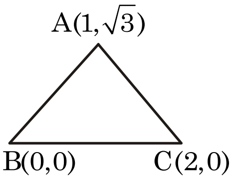
$\Rightarrow \sin\theta\cos\theta = \dfrac{3}{8}$

$\Rightarrow \sin\theta - \cos\theta = \dfrac{4}{3} \times \dfrac{3}{8} = \dfrac{1}{2}$

51. Centroid $= \left(1, \dfrac{7}{3}\right)$

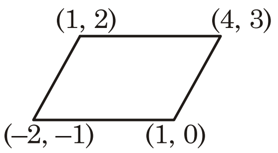


52. ΔABC is equilateral



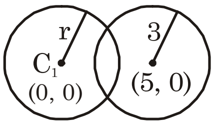
$\Rightarrow$ Incentre = Centroid = $\left(1, \frac{1}{\sqrt{3}}\right)$

53.



4^{th} vertex = (4 – 2 – 1, 3 – 1 – 0) = (1, 2)

54.



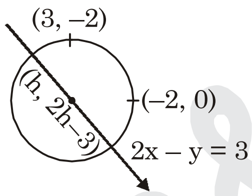
$r - 3 < C_1C_2 < r + 3$

$\Rightarrow r - 3 < 5 < r + 3$

$\Rightarrow r > 2, r < 8$

$\Rightarrow 2 < r < 8$

55. $(h - 3)^2 + (2h - 1)^2 = (h + 2)^2 + (2h - 3)^2$



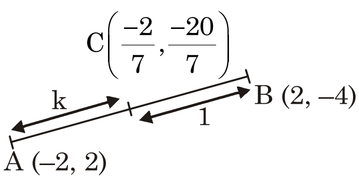
$\Rightarrow h = -\frac{3}{2} \Rightarrow 2h - 3 = -6$

$\therefore r^2 = \frac{1}{4} + 36$

Equation of circle is $\left(x + \frac{3}{2}\right)^2 + (y + 6)^2 = \frac{1}{4} + 36$

$\Rightarrow x^2 + y^2 + 3x + 12y + 2 = 0$

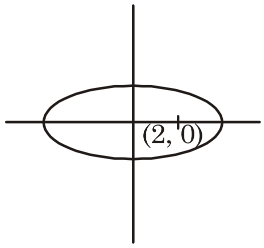
56. $\frac{2k - 2}{k + 1} = -\frac{2}{7}$



$\Rightarrow 7(k - 1) = -k - 1$

$\Rightarrow 8k = 6 \Rightarrow k = \frac{3}{4}$ ratio = 3 : 4

57. $2 = \frac{a}{4} \Rightarrow a = 8$



$b^2 = 64 - 4 = 60$

Equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{60} = 1$

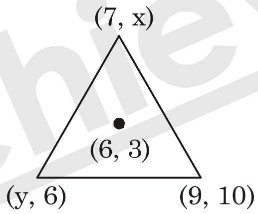
58. Let equation of line is $2x + 3y + \lambda = 0$

putting (–1, 2), we get $\lambda = -4$

$\Rightarrow$ equation of line is $2x + 3y - 4 = 0$

59. $\theta = \tan^{-1}\left(\frac{\frac{-\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}}}{1 + \left(\frac{-\sqrt{2}}{\sqrt{3}}\right)\left(\frac{-\sqrt{3}}{\sqrt{2}}\right)}\right) = \tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$

60.



$16 + y = 18 \Rightarrow y = 2$

Next, $x + 4 = 9 \Rightarrow x = 5$

$\therefore (x, y) = (5, 2)$

61. Parallel to y-axis.

62. Let O (0, 0, 0) A (a, 0, 0), B (0, b, 0) and C (0, 0, c).

$P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from O, A, B and C.

63. P, Q, R, S are collinear.

64. Let $\frac{x - 1}{2} = \frac{y - 2}{-3} = \frac{z + 1}{3} = \lambda$

$\Rightarrow x = 2\lambda + 1, y = -3\lambda + 2, z = 3\lambda - 1$

putting $x = 0$, $\lambda = -\frac{1}{2}$

$\therefore y = \frac{7}{2}, z = -\frac{5}{2}$

So, point $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

65. $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$ ...(i)

$\frac{x-f}{e} = \frac{y-0}{1} = \frac{z-h}{g}$ ...(ii)

(i) and (ii) are perpendicular

$\Rightarrow$ ae + 1 + cg = 0

66. $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$

$\Rightarrow$ 3n – 2m + 2n – 2 + 2m – 3 = 0

$\Rightarrow$ n = 1

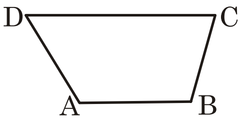
Next, $1 + m^2 + n^2 = 6 \Rightarrow m = \pm 2$

67. $\overrightarrow{OA} + \overrightarrow{OC} = 2\left(\overrightarrow{OP}\right)$ .... (1)

$\overrightarrow{OB} + \overrightarrow{OD} = 2\left(\overrightarrow{OP}\right)$ .... (2)

adding (1) and (2) we get,

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$

68. 

$\overrightarrow{BA} + \overrightarrow{AD} = \overrightarrow{BD}$ ...(i)

$\overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{CA}$ ...(ii)

adding (i) and (ii)

$\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} + \overrightarrow{CA}$

69. $\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{b} \times \vec{c} = \vec{a} \Rightarrow \vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ form an orthogonal system.

Next, $|\vec{a} \times \vec{b}| = |\vec{c}| \Rightarrow |\vec{a}|\,|\vec{b}| \sin 90° \,.\, 1 = |\vec{c}|$

and $|\vec{b} \times \vec{c}| = \vec{a} \Rightarrow |\vec{b}|\,|\vec{c}| = \sin 90° \,.\, 1 = |\vec{a}|$

putting for $|\vec{c}|$, we get $|\vec{b}||\vec{a}||\vec{b}| = |\vec{a}| \Rightarrow |\vec{b}| = 1$

and $|\vec{a}| = |\vec{c}|$

70. (2) (3) + (3) (2) – 4λ = 0 $\Rightarrow \lambda = 3$

71. $\lim_{x \to 0} \frac{e^x - (1+x)}{x^2}$

$= \lim_{x \to 0} \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots..\right) - (1+x)}{x^2} = \frac{1}{2!} = \frac{1}{2}$

72. $\int_0^{\pi/2} \frac{1}{1+\cos\theta} d\theta$

$= \int_0^{\pi/2} \frac{1}{2\cos^2\frac{\theta}{2}} d\theta = \frac{1}{2} \int_0^{\pi/2} \sec^2 \frac{\theta}{2} d\theta$

$= \frac{1}{2} \times 2 \left[\tan \frac{\theta}{2}\right]_0^{\pi/2} = 1$

73. $\int \frac{1}{x(x^7+1)} dx$

$= \frac{1}{7} \int \frac{7x^6}{x^7(x^7+1)} dx$

putting $x^7 = t$

$= \frac{1}{7} \int \frac{7x^6}{t(t+1)} dt = \frac{1}{7} \int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$

$= \frac{1}{7}$ [log t – log (t + 1)] + c

$= \frac{1}{7} \log\left(\frac{t}{t+1}\right) + c = \frac{1}{7} \log\left(\frac{x^7}{x^7+1}\right) + c$

74. f(x) = cosx is bijective for domain X = [0, π] and co-domain Y = [–1 , 1].

75. $\frac{f(a)}{f(a+1)} = \frac{\frac{a}{a-1}}{\frac{a+1}{a}} = \frac{a}{a-1} \times \frac{a}{a+1} = \frac{a^2}{a^2-1} = f(a^2)$

76. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx$

$= \frac{1}{e} \int \frac{ex^{e-1} + e^x}{x^e + e^x} dx$

$= \frac{1}{e} \log (x^e + e^x) + c$

77. $f(x) = x^2 - 3$

f o f (x) = $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$

f o f o f (x) = $(x^4 - 6x^2 + 6)^2 - 3$

f o f o f (x) is even function

$\Rightarrow$ (f o f o f) (–1) = (f o f o f) (1)

Next (f o f o f) (–1) –4 (f o f o f) (1)

–3 {(f o f o f) (1)} = (–3) (–2) = 6

f o f (o) = 6

Both 1 and 2 are correct.

78. p(mx + n) + q = m(px + q) + n

$\Rightarrow$ pn + q = qm + n

$\Rightarrow$ f(n) = g(q)

79. $\underset{x\to 1}{\mu m}\ \dfrac{F(x)-F(1)}{x-1}$

$= \{F'(x)\}_{x=1} = \left(\dfrac{-2x}{2\sqrt{9-x^2}}\right)_{x=1} = \dfrac{-1}{2\sqrt{2}}$

80. $\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{1}{\left(\dfrac{dy}{dx}\right)}\right)$

$= \dfrac{-\dfrac{d}{dy}\left(\dfrac{dy}{dx}\right)}{\left(\dfrac{dy}{dx}\right)^2} = \dfrac{-\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)\times\dfrac{dx}{dy}}{\left(\dfrac{dy}{dx}\right)^2}$

$= -\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

81. $(f-g)(x) = \begin{cases} x\ , & x \in Q \\ -x\ , & x \in R-Q \end{cases}$

clearly $f-g$ is one-one and on to

82. $f(x) = \sin 3x$

$\sin 3x$ is increasing in

$\left[-\dfrac{\pi}{6}, \dfrac{\pi}{6}\right]$

Interval length $= \dfrac{\pi}{3}$

83. $x\,dy = ydx + y^2dy$

$\Rightarrow \int \dfrac{ydx - xdy}{y^2} = -\int dy \Rightarrow -y = \dfrac{x}{y} + C$

$y(1) = 1 \Rightarrow C = -2 \Rightarrow -y = \dfrac{x}{y} - 2$

$\therefore\ y(-3) = 3\ (\because\ y(x) > 0)$

84. Maximum value $= 4 + 1 = 5$

85. $f(x) = \int \sin^2 x dx = \int\left(\dfrac{1-\cos 2x}{2}\right)dx$

$= \dfrac{1}{2}\left[x - \dfrac{1}{2}\sin 2x\right] + C$

$f(x+\pi) \neq f(x)$ ($\because$ f(x) is not periodic)

Statement 1 false

Next, $\sin^2(\pi + x) = \sin^2 x$

Statement 2 true.

86. $y = x\left(\dfrac{dy}{dx}\right)^2 + \left(\dfrac{dy}{dx}\right)^{-2}$

$\Rightarrow y\left(\dfrac{dy}{dx}\right)^2 = x\left(\dfrac{dy}{dx}\right)^4 + 1$

order = 1, degree = 4

87. $y^2 - 2ay + x^2 = a^2$

$\Rightarrow 2y\dfrac{dy}{dx} - 2a\dfrac{dy}{dx} + 2x = 0 \Rightarrow \dfrac{y\dfrac{dy}{dx} + x}{\dfrac{dy}{dx}} = a$

$\Rightarrow \dfrac{py+x}{p} = a \Rightarrow y^2 \dfrac{-2y(py+x)}{p} + x^2 = \left(\dfrac{py+x}{p}\right)^2$

$\Rightarrow p^2y^2 - 2p^2y^2 - 2xyp + p^2x^2 = p^2y^2 + x^2 + 2xyp$

$\Rightarrow -2p^2y^2 + p^2x^2 - 4xyp - x^2 = 0$

$\Rightarrow p^2(x^2 - 2y^2) - 4xyp - x^2 = 0$

88. $ydx = (x + 2y^2)dy$

$\Rightarrow \dfrac{dx}{xy} - \dfrac{x}{y} - 2y$

I.F $= e^{\int -\frac{1}{y}dy} = -\dfrac{1}{y}$

Solution is given by, $-\dfrac{x}{y} = \int -2dy$

$\Rightarrow \dfrac{x}{y} = 2y + c \quad \Rightarrow x = 2y^2 + cy$

89. $f(x+y) = f(x).\,f(y)\ \forall\ x,y \in R$

$\Rightarrow f(0+0) = f(0).f(0) \Rightarrow \{f(0)\}^2 - f(0) = 0 \Rightarrow f(0) = 1$

Next, $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(x).f(h)-f(x)}{h}$

$= f(x)\cdot \lim\limits_{h\to 0}\dfrac{f(h)-1}{h}$

$= f(x) \lim\limits_{h\to 0}\dfrac{f(h)-f(o)}{h} = f(x).\,f'(o)$

$\therefore f'(5) = f(5)\ .\ f'(o).$

90. $I = \int_0^a f(x)\,g(x)dx = \int_0^a f(a-x).g(a-x)dx$

$= \int_0^a f(x)\{2 - g(x)\}dx = \int_0^a 2f(x)\,dx - \int_0^a f(x).g(x)\,dx$

$\Rightarrow 2I = 2\int_0^a f(x)dx \Rightarrow I = \int_0^a f(x)dx$

91. $\log\left(\frac{dy}{dx}\right) = a \Rightarrow \frac{dy}{dx} = e^a \Rightarrow \int dy = \int e^a\, dx$

$\Rightarrow y = x\, e^a + c$

92. $f(x) = \begin{cases} 2x+1, -3 < x < -2 \\ x-1, -2 \le x < 0 \\ x+2, 0 \le x < 1 \end{cases}$

Here $f(0^-) = -1$ and $f(0^+) = 2$ clearly f(x) is discontinuous at x = 0 and continuous at all other points

93. If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exists, then $\lim_{x\to a} f(x).g(x)$ exists. But if $\lim_{x\to a} f(x).g(x)$ exists, then it is not necessery that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exists.

94. If $f(x) = x^2(x-3)$ then $f(-x) = x^2(-x-3) \ne f(x)$ or $-f(x)$

$\Rightarrow$ f(x) is neither even nor odd.

95. $\frac{d}{dx} \log_{10}^{(5x^2+3)}$

$= \frac{1}{5x^2+3} \times \log_{10}^{e} \times 10x = \frac{10x \log_{10}^{e}}{5x^2+3}$

96. $f(a) = \frac{a-1}{a+1}$

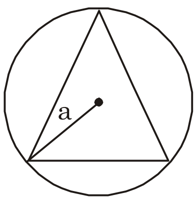
$f(2a) = \frac{2a-1}{2a+1}$

$f(a) + 1 = \frac{a-1}{a+1} + 1 = \frac{2a}{a+1}$

So, $f(2a) \ne f(a) + 1$

Next, $f\left(\frac{1}{a}\right) = \frac{\frac{1}{a}-1}{\frac{1}{a}+1} = \frac{1-a}{a+1} = -\left(\frac{a-1}{a+1}\right) = -f(a)$

97. For area of Δ to be maximum, triangle should be an equilateral triangle.



ℓ = length of side of equilateral triangle $= \sqrt{3}a$

$\therefore$ Area $= \frac{\sqrt{3}}{4}\left(\sqrt{3}\,a\right)^2 = \frac{3\sqrt{3}}{4}a^2$

98. $f(x) = x + \frac{1}{x} \Rightarrow f'(x) = \frac{x^2-1}{x^2} \Rightarrow f'(x) = \frac{(x-1)(x+1)}{x^2}$

for $x \in (0, 1)$, $f'(x) < 0$

$\Rightarrow$ f(x) decreases

99. $f(x) = x^n$, $n \ne 0$.

$\Rightarrow f'(x) = n\,x^{n-1}$

f(x) to be differentiable, $n - 1 \ge 0 \Rightarrow n \ge 1$

$\Rightarrow n \in [1, \infty]$

100. $\int_{e^{-1}}^{e^2} \left|\frac{\log x}{x}\right| dx = \int_{e^{-1}}^{e^0} -\frac{\log x}{x}\, dx + \int_{e^0}^{e^2} \frac{\log x}{x}\, dx$

$= -\frac{1}{2}\left[(\log x)^2\right]_{e^{-1}}^{e^0} + \frac{1}{2}\left[(\log x)^2\right]_{e^0}^{e^2} = \frac{1}{2} + 2 = \frac{5}{2}$

101. New variance $= 5 \times (3)^2 = 45$

102. Required mean

$= \frac{20\times100 - (21+21+18+20)}{96} = \frac{1920}{96} = 20$

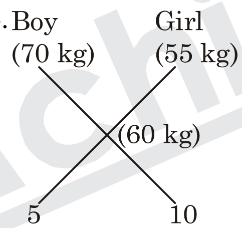
103. Required probability

$= \frac{{}^2C_0 \times {}^2C_2}{{}^4C_2} = \frac{1}{6}$

104. Required probability

$= 1 - \left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

105.



By aligation, ratio = 1 : 2

$\therefore$ no. of boys $= \frac{1}{3} \times 150 = 50$

106. $A \subseteq B \Rightarrow A \cap B = A$

$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$

107. Required probability

$= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$

108. $r = \sqrt{\frac{15}{4}\times\frac{2}{30}} = \frac{1}{2}$

109. Angle $= \frac{2}{5} \times 360° = 144°$

110. It is a fundamental concept. So, options 'a' is correct.

111. $P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$

112. $S.D = \frac{5}{4}$ M.D. (formula)

113. Data can be represented in tabular and graphical form.

114. The abscissa of the point of intersection of less than and more than ogive is median.

115. Both statements are correct

116. Result $= \sqrt{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{8}\right)} = -\frac{1}{4}$

117. Median remains same but the mean will decrease.

118. Required probability

$$= 1 - \frac{9}{36} = 1 - \frac{1}{4} = \frac{3}{4}$$

119. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$$= 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{5}{12}$$

120. $np = 12$ and $npq = 4$

$$\Rightarrow q = \frac{4}{12} = \frac{1}{3} \Rightarrow p = \frac{2}{3}$$

So, $n \times \frac{2}{3} = 12 \Rightarrow n = 18$