

Mathematics

1. $\tan 18^\circ = \frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ (Result)

2. $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$\Rightarrow \tan^{-1} \frac{2y}{1-y^2} = \tan^{-1} \frac{x+z}{1-xz}$

$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz} = \frac{x+z}{1-y^2} (\because y^2 = xz)$

$\Rightarrow 2y = x + z$

$\Rightarrow x, y, z \in A.P$

But $x, y, z \in G.P$ (given)

$\Rightarrow x = y = z$

3. $\tan 2\alpha = \tan (\alpha + \beta + \alpha - \beta)$

$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{2+1}{1-2 \times 1} = \frac{3}{-1} = -3$

4. $A + B + C = \pi$

$\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \cos \frac{A}{2}$ (True)

$\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{A}{2}\right) = \cot \frac{A}{2}$ (True)

Statement 1 and 2 are true.

5. $\sec \theta - \operatorname{cosec} \theta = \frac{4}{3}$

$\Rightarrow \frac{\sin \theta - \cos \theta}{\sin \theta \cos \theta} = \frac{4}{3}$

$\Rightarrow \frac{(\sin \theta - \cos \theta)^2}{(\sin \theta \cos \theta)^2} = \frac{16}{9}$

$\Rightarrow \frac{1 - 2\sin \theta \cos \theta}{\sin^2 \theta \cos^2 \theta} = \frac{16}{9}$

Let $\sin \theta \cos \theta = x$

$\Rightarrow \frac{1-2x}{x^2} = \frac{16}{9}$

$\Rightarrow 16x^2 + 18x - 9 = 0$

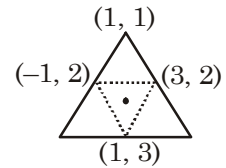
$\Rightarrow (8x - 3)(2x + 3) = 0$

$\Rightarrow x = \frac{3}{8}, x = -\frac{3}{2}$

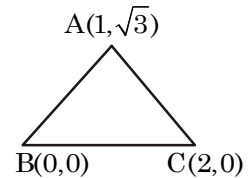
$\Rightarrow \sin \theta \cos \theta = \frac{3}{8}$

$\Rightarrow \sin \theta - \cos \theta = \frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$

6. Centroid = $\left(1, \frac{7}{3}\right)$

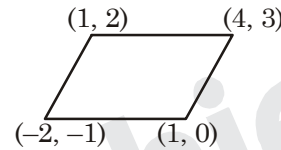


7. ΔABC is equilateral



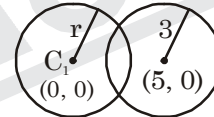
\Rightarrow Incentre = Centroid = $\left(1, \frac{1}{\sqrt{3}}\right)$

8.



4th vertex = $(4 - 2 - 1, 3 - 1 - 0) = (1, 2)$

9.



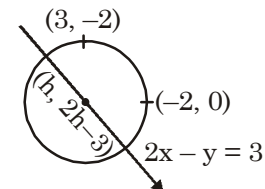
$r - 3 < C_1 C_2 < r + 3$

$\Rightarrow r - 3 < 5 < r + 3$

$\Rightarrow r > 2, r < 8$

$\Rightarrow 2 < r < 8$

10. $(h - 3)^2 + (2h - 1)^2 = (h + 2)^2 + (2h - 3)^2$



$\Rightarrow h = -\frac{3}{2} \Rightarrow 2h - 3 = -6$

$\therefore r^2 = \frac{1}{4} + 36$

Equation of circle is $\left(x + \frac{3}{2}\right)^2 + (y + 6)^2 = \frac{1}{4} + 36$

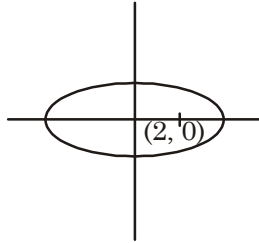
$\Rightarrow x^2 + y^2 + 3x + 12y + 2 = 0$

11. $\frac{2k-2}{k+1} = -\frac{2}{7}$

$\Rightarrow 7(k-1) = -k-1$

$\Rightarrow 8k = 6 \Rightarrow k = \frac{3}{4}$ ratio = 3 : 4

12. $2 = \frac{a}{4} \Rightarrow a = 8$

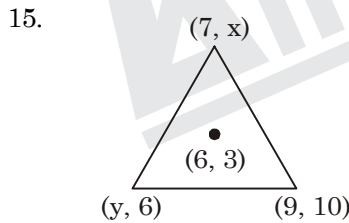


$b^2 = 64 - 4 = 60$

Equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{60} = 1$

13. Let equation of line is $2x + 3y + \lambda = 0$
 putting $(-1, 2)$, we get $\lambda = -4$
 \Rightarrow equation of line is $2x + 3y - 4 = 0$

14. $\theta = \tan^{-1} \left(\frac{\frac{-\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}}}{1 + \left(\frac{-\sqrt{2}}{\sqrt{3}}\right)\left(\frac{-\sqrt{3}}{\sqrt{2}}\right)} \right) = \tan^{-1} \left(\frac{1}{2\sqrt{6}} \right)$



$16 + y = 18 \Rightarrow y = 2$

Next, $x + 4 = 9 \Rightarrow x = 5$

$\therefore (x, y) = (5, 2)$

16. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = 1(9-15) - 2(18-15) + 3(10-5)$

$= -6 - 6 + 15 = 3 \neq 0$

\Rightarrow system has unique solution.

17. $\begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x+y \\ x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

solving we get, $x = 2, y = -2$

$A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

18. $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$\begin{vmatrix} 1 & 1 & 1 \\ 0 & xyz & 0 \\ 0 & 0 & xyz \end{vmatrix} = x^2y^2z^2$

19. $x^3 + y^3 = 0 \Rightarrow x^2 - xy + y^2 = 0$

$\Rightarrow \left(\frac{x}{y}\right)^2 - \frac{x}{y} + 1 = 0, \frac{x}{y} = -1$

$\Rightarrow \frac{x}{y}$ is one of the cube roots of -1

20. By symmetry B has as many elements as C.

21. $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

22. $(1 \times 3)(3 \times 3)(3 \times 1) = (1 \times 1)$

23. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^4 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

24. $\sin A = \frac{3}{5}$

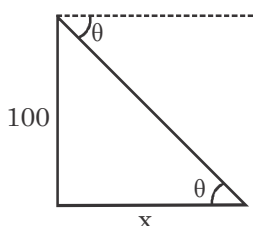
$\Rightarrow \cos A = -\frac{4}{5}$ (\because A lies in 2nd quadrant)

$\therefore \cos^2 \frac{A}{2} = \frac{1 + \cos A}{2} = \frac{1}{10}$

$\Rightarrow \cos \frac{A}{2} = \frac{1}{\sqrt{10}}$ ($\frac{A}{2}$ lies in 1st Quadrant)

$$\begin{aligned}
 25. \quad & \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} \\
 &= \frac{\cos 10^\circ - \sqrt{3} \sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ} \\
 &= \frac{2 \times 2 \left[\frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ \right]}{2 \sin 10^\circ \cos 10^\circ} \\
 &= 4 \frac{\cos(60^\circ + 18^\circ)}{\sin 20^\circ} = 4 \frac{\cos 70^\circ}{\sin 20^\circ} = 4
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \tan \theta = \frac{5}{12} \\
 \Rightarrow & \frac{100}{x} = \frac{5}{12} \\
 \Rightarrow & x = 240 \text{ m}
 \end{aligned}$$



$$\begin{aligned}
 27. \quad & f(x) = \sin \left(x + \frac{\pi}{6} \right) + \cos \left(x + \frac{\pi}{6} \right) = \sqrt{2} \sin \left(x + \frac{\pi}{6} + \frac{\pi}{4} \right) \\
 & f(x) \text{ is maximum when } x + \frac{\pi}{6} + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{12}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & K = \sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ \\
 &= \frac{1}{4} \sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} \\
 &= \frac{2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)}{2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)} = \tan \left(\frac{\alpha + \beta}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{\sin(\theta + 2\alpha)}{\sin \theta} = \frac{1}{3} \\
 \Rightarrow & \frac{\sin(\theta + 2\alpha) + \sin \theta}{\sin(\theta + 2\alpha) - \sin \theta} = \frac{4}{-2} = -2 \\
 \Rightarrow & \frac{2 \sin(\theta + \alpha) \cos \alpha}{2 \cos(\theta + \alpha) \sin \alpha} = -2 \\
 \Rightarrow & \frac{\tan(\theta + \alpha)}{\tan \alpha} = -2 \\
 \Rightarrow & \tan(\theta + \alpha) + 2 \tan \alpha = 0
 \end{aligned}$$

$$\begin{aligned}
 31. \quad & -b = \frac{b^2 - 2c}{c^2} \\
 \Rightarrow & -bc^2 = b^2 - 2c \Rightarrow 2c = b^2 + bc^2 \\
 \Rightarrow & \frac{2c}{b} = b + c^2 \Rightarrow \frac{2}{b} = \frac{b}{c} + c \\
 \Rightarrow & c, \frac{1}{b}, \frac{b}{c} \in \text{A.P.} \Rightarrow \frac{1}{c}, b, \frac{c}{b} \in \text{H.P.}
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & -\frac{1}{a} = \frac{\frac{1}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} \\
 \Rightarrow & -\frac{1}{a} = \frac{1 - 2ca}{c^2} \\
 \Rightarrow & a - 2ca^2 = -c^2 \\
 \Rightarrow & 2ca^2 = a + c^2 \\
 \Rightarrow & a, ca^2, c^2 \in \text{A.P.}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & {}^8C_1 + {}^8C_2 + \dots + {}^8C_7 \\
 &= 2^8 - 2 = 254
 \end{aligned}$$

$$34. \quad {}^3P_2 \times 6! = 6 \times 720 = 4320$$

$$\begin{aligned}
 35. \quad & a_n = 2n - 3 \\
 \Rightarrow & a_5 = 7
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & 11 + 13 + \dots + 99 \\
 &= \frac{45}{2} (11 + 99) = 45 \times 55 = 2475
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots \\
 &= \left(1 - \frac{1}{2} \right) + \left(1 - \frac{1}{4} \right) + \left(1 - \frac{1}{8} \right) + \dots \\
 &= n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) \\
 &= n - \frac{1}{2} \left(1 - \frac{1}{2^n} \right) = n - 1 + 2^{-n} \\
 & \qquad \qquad \qquad \frac{1}{2}
 \end{aligned}$$

38. By property, $(A - B) \cup A = A$, $(A - B) \cap B = \phi$ and $A \subseteq B \Rightarrow A \cup B = B$ are true.

$$\begin{aligned}
 39. \quad & (1p101)_2 + (10q1)_2 = (100r00)_2 \\
 & \text{Equating, } P = 0, q = 1, r = 0
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & S = \{x : x^2 + 1 = 0, x \in \mathbb{R}\} \\
 \Rightarrow & S = \phi
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & {}^nC_4 x^{n-4} y^4 - {}^nC_5 x^{n-5} y^5 = 0 \\
 \Rightarrow & {}^nC_4 x^{n-4} y^4 = {}^nC_5 x^{n-5} y^5 \\
 \Rightarrow & \frac{x}{y} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}
 \end{aligned}$$

42. $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} \alpha^2 + 4 & 4\alpha \\ 4\alpha & \alpha^2 + 4 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \alpha^3 + 12\alpha & 6\alpha^2 + 8 \\ 6\alpha^2 + 8 & \alpha^3 + 12\alpha \end{bmatrix}$

$\Rightarrow |A^3| = 125$

$\Rightarrow (\alpha^3 + 12\alpha)^2 - (6\alpha^2 + 8)^2 = 125$

putting $\alpha = \pm 3$, $(63)^2 - (62)^2 = 125$ satisfied.

43. $|B^{-1}AB| = |B^{-1}| |A| |B|$
 $= \frac{1}{|B|} |A| |B| = |A|$

44. Putting $x = 0$

$\begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$ is a skew-symmetric matrix of odd order

whose determinant value is zero.

45. $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$AA^T = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{Identity matrix}$

46. $R = \{(x, y) : x \text{ and } y \text{ were born in Delhi on same day}\}$
 R is an equivalence relation.

47. Required no. of subsets
 $= {}^{10}C_2 + {}^{10}C_3 = 45 + 120 = 165$

48. $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3} = 0$

49. $(\alpha - \beta)^2 < 5$
 $\Rightarrow k^2 - 4 < 5 \Rightarrow k^2 < 9$
 $\Rightarrow -3 < k < 3$ (1)

Next, $|k| \geq 2 \Rightarrow k \leq -2$ or $k \geq 2$ (2)
 from both, $k \in (-3, -2] \cup [2, 3)$

50. Let $\alpha, k\alpha$ be roots of $x^2 + px + q = 0$
 and $\beta, k\beta$ be roots of $x^2 + \ell x + m = 0$

Clearly, $\frac{\alpha^2}{\beta^2} = \frac{p^2}{\ell^2} = \frac{q}{m} \Rightarrow p^2 m = \ell^2 q$

51. $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n = \omega^n + \omega^{2n} = -1$, n is

not multiple of 3

52. Sum = $12(10^0 + 10^1 + 10^2)$
 $= 111 \times 12 = 1332$

53. $0.3 + 0.33 + 0.333 + \dots$

$= \frac{3}{10} + \frac{33}{100} + \frac{333}{1000} + \dots$

$= \frac{3}{9} \left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \right]$

$= \frac{1}{3} \left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \dots \right]$

$= \frac{1}{3} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n}\right) \right] = \frac{1}{3} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n}\right) \right]$

54. $(1 + \omega)(1 + \omega^2)(1 + \omega^3)(1 + \omega + \omega^2) = 0$

55. In an A.P
 If $S_m = n$ and $S_n = m$ then $S_{m+n} = -(m+n)$

56. $\frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-(-2i)} = \frac{1+2i}{1+2i} = 1$

Modulus = 1, Argument = 0

57. If graph of quadratic lies entirely above x-axis then $D > 0$. So both roots are complex.

58. $|z+1| = |z+4-3|$
 $\leq |z+4| + |-3|$
 maximum value of $|z+1| = 6$

59. $z^2 = 2\bar{z}$
 Let $z = x + iy$
 Now $x^2 - y^2 + 2xyi = 2x - 2yi$
 $\Rightarrow x = -1$ and $y = \pm\sqrt{3}$

So, $z = -1 + \sqrt{3}i$ and $z = -1 - \sqrt{3}i$
 are two roots

60. $\cot(\alpha + \beta) = \frac{\cot \alpha - \cot \beta - 1}{\cot \beta + \cot \alpha}$
 $= \frac{c-1}{-b} = \frac{1-c}{b}$

61. $A \subset B \Rightarrow A \cap B = A$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$

62. Required probability

$= \frac{\pi \left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$

63. $r = \sqrt{\frac{15}{4} \times \frac{2}{30}} = \frac{1}{2}$

64. Angle = $\frac{2}{5} \times 360^\circ = 144^\circ$

65. It is a fundamental concept. So, options 'a' is correct.

66. $P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$

67. $S.D = \frac{5}{4} \text{ M.D. (formula)}$

68. Data can be represented in tabular and graphical form.

69. The abscissa of the point of intersection of less than and more than ogive is median.

70. Both statements are correct

71. Result = $\sqrt{\left(-\frac{1}{2}\right) \times \left(-\frac{1}{8}\right)} = -\frac{1}{4}$

72. Median remains same but the mean will decrease.

73. Required probability

$$= 1 - \frac{9}{36} = 1 - \frac{1}{4} = \frac{3}{4}$$

74. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$$= 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{5}{12}$$

75. $np = 12$ and $npq = 4$

$$\Rightarrow q = \frac{4}{12} = \frac{1}{3} \Rightarrow p = \frac{2}{3}$$

$$\text{So, } n \times \frac{2}{3} = 12 \Rightarrow n = 18$$

76. $\log\left(\frac{dy}{dx}\right) = a \Rightarrow \frac{dy}{dx} = e^a \Rightarrow \int dy = \int e^a dx$

$$\Rightarrow y = x e^a + c$$

$$77. f(x) = \begin{cases} 2x + 1, & -3 < x < -2 \\ x - 1, & -2 \leq x < 0 \\ x + 2, & 0 \leq x < 1 \end{cases}$$

Here $f(0^-) = -1$ and $f(0^+) = 2$ clearly $f(x)$ is discontinuous at $x = 0$ and continuous at all other points

78. If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exists, then $\lim_{x \rightarrow a} f(x).g(x)$

exists. But if $\lim_{x \rightarrow a} f(x).g(x)$ exists, then it is not necessary

that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exists.

79. If $f(x) = x^2(x - 3)$ then $f(-x) = x^2(-x - 3) \neq f(x)$ or $-f(x)$
 $\Rightarrow f(x)$ is neither even nor odd.

80. $\frac{d}{dx} \log_{10}(5x^2 + 3)$

$$= \frac{1}{5x^2 + 3} \times \log_{10}^e \times 10x = \frac{10x \log_{10}^e}{5x^2 + 3}$$

81. $f(a) = \frac{a-1}{a+1}$

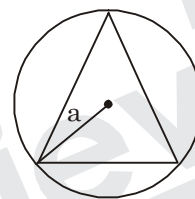
$$f(2a) = \frac{2a-1}{2a+1}$$

$$f(a) + 1 = \frac{a-1}{a+1} + 1 = \frac{2a}{a+1}$$

So, $f(2a) \neq f(a) + 1$

$$\text{Next, } f\left(\frac{1}{a}\right) = \frac{\frac{1}{a}-1}{\frac{1}{a}+1} = \frac{1-a}{a+1} = -\left(\frac{a-1}{a+1}\right) = -f(a)$$

82. For area of Δ to be maximum, triangle should be an equilateral triangle.



ℓ = length of side of equilateral triangle = $\sqrt{3}a$

$$\therefore \text{Area} = \frac{\sqrt{3}}{4} (\sqrt{3} a)^2 = \frac{3\sqrt{3}}{4} a^2$$

83. $f(x) = x + \frac{1}{x} \Rightarrow f'(x) = \frac{x^2-1}{x^2}$

$$\Rightarrow f'(x) = \frac{(x-1)(x+1)}{x^2}$$

for $x \in (0, 1)$, $f'(x) < 0$

$\Rightarrow f(x)$ decreases

84. $f(x) = x^n, n \neq 0.$

$$\Rightarrow f'(x) = n x^{n-1}$$

$f(x)$ to be differentiable, $n - 1 \geq 0 \Rightarrow n \geq 1$

$$\Rightarrow n \in [1, \infty]$$

85. $\int_{e^{-1}}^{e^2} \left| \frac{\log x}{x} \right| dx = \int_{e^{-1}}^{e^0} -\frac{\log x}{x} dx + \int_{e^0}^{e^2} \frac{\log x}{x} dx$

$$= -\frac{1}{2} [(\log x)^2]_{e^{-1}}^{e^0} + \frac{1}{2} [(\log x)^2]_{e^0}^{e^2} = \frac{1}{2} + 2 = \frac{5}{2}$$

86. New variance = $5 \times (3)^2 = 45$

87. Required mean

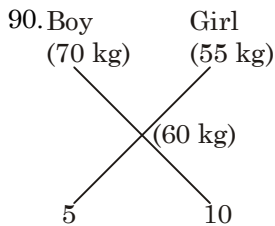
$$= \frac{20 \times 100 - (21 + 21 + 18 + 20)}{96} = \frac{1920}{96} = 20$$

88. Required probability

$$= \frac{{}^2C_0 \times {}^2C_2}{{}^4C_2} = \frac{1}{6}$$

89. Required probability

$$= 1 - \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \right) = 1 - \frac{1}{4} = \frac{3}{4}$$



By allegation, ratio = 1 : 2

$$\therefore \text{no. of boys} = \frac{1}{3} \times 150 = 50$$

91. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx$

$$= \frac{1}{e} \int \frac{ex^{e-1} + e^x}{x^e + e^x} dx$$

$$= \frac{1}{e} \log(x^e + e^x) + c$$

92. $f(x) = x^2 - 3$

$$f \circ f(x) = (x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$$

$$f \circ f \circ f(x) = (x^4 - 6x^2 + 6)^2 - 3$$

$f \circ f \circ f(x)$ is even function

$$\Rightarrow (f \circ f \circ f)(-1) = (f \circ f \circ f)(1)$$

$$\text{Next } (f \circ f \circ f)(-1) - 4(f \circ f \circ f)(1)$$

$$-3 \{(f \circ f \circ f)(1)\} = (-3)(-2) = 6$$

$$f \circ f(0) = 6$$

Both 1 and 2 are correct.

93. $p(mx + n) + q = m(px + q) + n$

$$\Rightarrow pn + q = qm + n$$

$$\Rightarrow f(n) = g(q)$$

94. $\lim_{x \rightarrow 1} \frac{F(x) - F(1)}{x - 1}$

$$= \{F'(x)\}_{x=1} = \left(\frac{-2x}{2\sqrt{9-x^2}} \right)_{x=1} = \frac{-1}{2\sqrt{2}}$$

95. $\frac{d^2x}{dy^2} = \frac{d}{dy} \left(\frac{1}{\left(\frac{dy}{dx}\right)} \right) = \frac{-\frac{d}{dy} \left(\frac{dy}{dx}\right)}{\left(\frac{dy}{dx}\right)^2} = \frac{-\frac{d}{dx} \left(\frac{dy}{dx}\right) \times \frac{dx}{dy}}{\left(\frac{dy}{dx}\right)^2}$

$$= -\left(\frac{d^2y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-3}$$

96. $(f - g)(x) = \begin{cases} x, & x \in Q \\ -x, & x \in R - Q \end{cases}$

clearly $f - g$ is one-one and on to

97. $f(x) = \sin 3x$

$\sin 3x$ is increasing in

$$\left[-\frac{\pi}{6}, \frac{\pi}{6} \right]$$

$$\text{Interval length} = \frac{\pi}{3}$$

98. $x dy = y dx + y^2 dy$

$$\Rightarrow \int \frac{y dx - x dy}{y^2} = -\int dy \Rightarrow -y = \frac{x}{y} + C$$

$$y(1) = 1 \Rightarrow C = -2 \Rightarrow -y = \frac{x}{y} - 2$$

$$\therefore y(-3) = 3 (\because y(x) > 0)$$

99. Maximum value = 4 + 1 = 5

100. $f(x) = \int \sin^2 x dx = \int \left(\frac{1 - \cos 2x}{2} \right) dx$

$$= \frac{1}{2} \left[x - \frac{1}{2} \sin 2x \right] + C$$

$f(x + \pi) \neq f(x)$ ($\because f(x)$ is not periodic)

Statement 1 false

$$\text{Next, } \sin^2(\pi + x) = \sin^2 x$$

Statement 2 true.

101. $y = x \left(\frac{dy}{dx} \right)^2 + \left(\frac{dy}{dx} \right)^{-2}$

$$\Rightarrow y \left(\frac{dy}{dx} \right)^2 = x \left(\frac{dy}{dx} \right)^4 + 1$$

order = 1, degree = 4

102. $y^2 - 2ay + x^2 = a^2$

$$\Rightarrow 2y \frac{dy}{dx} - 2a \frac{dy}{dx} + 2x = 0 \Rightarrow \frac{y \frac{dy}{dx} + x}{\frac{dy}{dx}} = a$$

$$\Rightarrow \frac{py + x}{p} = a \Rightarrow y^2 \frac{-2y(py + x)}{p} + x^2 = \left(\frac{py + x}{p} \right)^2$$

$$\Rightarrow p^2 y^2 - 2p^2 y^2 - 2xyp + p^2 x^2 = p^2 y^2 + x^2 + 2xyp$$

$$\Rightarrow -2p^2 y^2 + p^2 x^2 - 4xyp - x^2 = 0$$

$$\Rightarrow p^2(x^2 - 2y^2) - 4xyp - x^2 = 0$$

103. $y dx = (x + 2y^2) dy$

$$\Rightarrow \frac{dx}{xy} - \frac{x}{y} - 2y$$

$$I.F = \int -\frac{1}{y} dy = -\frac{1}{y}$$

Solution is given by, $-\frac{x}{y} = \int -2y dy$

$$\Rightarrow \frac{x}{y} = 2y + c \Rightarrow x = 2y^2 + cy$$

104. $f(x+y) = f(x) \cdot f(y) \forall x, y \in \mathbb{R}$
 $\Rightarrow f(0+0) = f(0) \cdot f(0) \Rightarrow \{f(0)\}^2 - f(0) = 0 \Rightarrow f(0) = 1$

Next, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{f(x) \cdot f(h) - f(x)}{h}$$

$$= f(x) \cdot \lim_{h \rightarrow 0} \frac{f(h) - 1}{h}$$

$$= f(x) \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = f(x) \cdot f'(0)$$

$$\therefore f'(5) = f(5) \cdot f'(0)$$

105. $I = \int_0^a f(x) g(x) dx$

$$= \int_0^a f(a-x) \cdot g(a-x) dx$$

$$= \int_0^a f(x) \{2 - g(x)\} dx$$

$$= \int_0^a 2f(x) dx - \int_0^a f(x) \cdot g(x) dx$$

$$\Rightarrow 2I = 2 \int_0^a f(x) dx$$

$$\Rightarrow I = \int_0^a f(x) dx$$

106. Parallel to y-axis.

107. Let O (0, 0, 0) A (a, 0, 0), B (0, b, 0) and C (0, 0, c).

P $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from O, A, B and C.

108. P, Q, R, S are collinear.

109. Let $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{3} = \lambda$

$$\Rightarrow x = 2\lambda + 1, y = -3\lambda + 2, z = 3\lambda - 1$$

putting $x = 0, \lambda = -\frac{1}{2}$

$$\therefore y = \frac{7}{2}, z = -\frac{5}{2}$$

So, point $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

110. $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} \dots (i)$

$$\frac{x-f}{e} = \frac{y-0}{1} = \frac{z-h}{g} \dots (ii)$$

(i) and (ii) are perpendicular

$$\Rightarrow ae + 1 + cg = 0$$

111. $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$

$$\Rightarrow 3n - 2m + 2n - 2 + 2m - 3 = 0$$

$$\Rightarrow n = 1$$

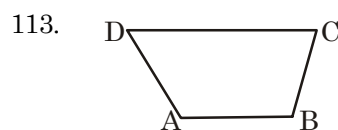
$$\text{Next, } 1 + m^2 + n^2 = 6 \Rightarrow m = \pm 2$$

112. $\vec{OA} + \vec{OC} = 2(\vec{OP}) \dots (1)$

$$\vec{OB} + \vec{OD} = 2(\vec{OP}) \dots (2)$$

adding (1) and (2) we get,

$$\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OP}$$



$$\vec{BA} + \vec{AD} = \vec{BD} \dots (i)$$

$$\vec{CD} + \vec{DA} + \vec{CA} \dots (ii)$$

adding (i) and (ii)

$$\vec{BA} + \vec{CD} = \vec{BD} + \vec{CA}$$

114. $\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{c}$ is perpendicular to both \vec{a} and \vec{b}

$\vec{b} \times \vec{c} = \vec{a} \Rightarrow \vec{a}$ is perpendicular to both \vec{b} and \vec{c}

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ form an orthogonal system.

$$\text{Next, } |\vec{a} \times \vec{b}| = |\vec{c}| \Rightarrow |\vec{a}| |\vec{b}| \sin 90^\circ \cdot 1 = |\vec{c}|$$

$$\text{and } |\vec{b} \times \vec{c}| = |\vec{a}| \Rightarrow |\vec{b}| |\vec{c}| \sin 90^\circ \cdot 1 = |\vec{a}|$$

$$\text{putting for } |\vec{c}|, \text{ we get } |\vec{b}| |\vec{a}| |\vec{b}| = |\vec{a}| \Rightarrow |\vec{b}| = 1$$

$$\text{and } |\vec{a}| = |\vec{c}|$$

$$115. (2) (3) + (3) (2) - 4\lambda = 0 \Rightarrow \lambda = 3$$

$$116. \lim_{x \rightarrow 0} \frac{e^x - (1+x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(1+x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots) - (1+x)}{x^2} = \frac{1}{2!} = \frac{1}{2}$$

$$117. \int_0^{\pi/2} \frac{1}{1 + \cos \theta} d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2 \cos^2 \frac{\theta}{2}} d\theta = \frac{1}{2} \int_0^{\pi/2} \sec^2 \frac{\theta}{2} d\theta$$

$$= \frac{1}{2} \times 2 \left[\tan \frac{\theta}{2} \right]_0^{\pi/2} = 1$$

$$118. \int \frac{1}{x(x^7+1)} dx$$

$$= \frac{1}{7} \int \frac{7x^6}{x^7(x^7+1)} dx$$

putting $x^7 = t$

$$= \frac{1}{7} \int \frac{7x^6}{t(t+1)} dt = \frac{1}{7} \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt$$

$$= \frac{1}{7} [\log t - \log(t+1)] + c$$

$$= \frac{1}{7} \log \left(\frac{t}{t+1} \right) + c = \frac{1}{7} \log \left(\frac{x^7}{x^7+1} \right) + c$$

119. $f(x) = \cos x$ is bijective for domain $X = [0, \pi]$ and co-domain $Y = [-1, 1]$.

$$120. \frac{f(a)}{f(a+1)} = \frac{\frac{a}{a-1}}{\frac{a}{a+1}} = \frac{a}{a-1} \times \frac{a+1}{a} = \frac{a^2}{a^2-1} = f(a^2)$$



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