Mathematics

1. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = 1(9-15)-2(18-15)+3(10-5)$

$= -6 - 6 + 15 = 3 \neq 0$

$\Rightarrow$ system has unique solution.

2. $\begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x+y \\ x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

solving we get, $x = 2, y = -2$

$A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

3. $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\begin{vmatrix} 1 & 1 & 1 \\ 0 & xyz & 0 \\ 0 & 0 & xyz \end{vmatrix} = x^2y^2z^2$

4. $x^3 + y^3 = 0 \Rightarrow x^2 - xy + y^2 = 0$

$\Rightarrow \left(\frac{x}{y}\right)^2 - \frac{x}{y} + 1 = 0,\ \frac{x}{y} = -1$

$\Rightarrow \frac{x}{y}$ is one of the cube roots of -1

5. By symmetry B has as many elements as C.

6. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

7. $(1 \times 3)(3 \times 3)(3 \times 1)$
$= (1 \times 1)$

8. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^4 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9. $\sin A = \frac{3}{5}$

$\Rightarrow \cos A = -\frac{4}{5}$ ($\because$ A lies in 2nd quadrant)

$\therefore \cos^2\frac{A}{2} = \frac{1+\cos A}{2} = \frac{1}{10}$

$\Rightarrow \cos\frac{A}{2} = \frac{1}{\sqrt{10}}$ ($\frac{A}{2}$ lies in 1st Quadrant)

10. $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ}$

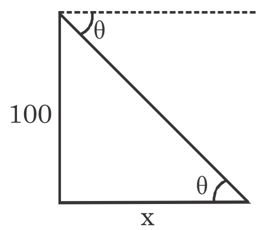
$= \frac{2 \times 2\left[\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right]}{2\sin 10^\circ \cos 10^\circ}$

$= 4\frac{\cos(60^\circ + 18)}{\sin 20^\circ} = 4\frac{\cos 70^\circ}{\sin 20^\circ} = 4$

11. $\tan\theta = \frac{5}{12}$

$\Rightarrow \frac{100}{x} = \frac{5}{12}$

$\Rightarrow x = 240$ m



12. $f(x) = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right) = \sqrt{2}\sin\left(x + \frac{\pi}{6} + \frac{\pi}{4}\right)$

f(x) is maximum when $x + \frac{\pi}{6} + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{12}$

13. $K = \sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ$

$= \frac{1}{4}\sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

14. $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$

$= \dfrac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$= \tan\left(\dfrac{\alpha+\beta}{2}\right)$

15. $\dfrac{\sin(\theta+2\alpha)}{\sin\theta} = \dfrac{1}{3}$

$\Rightarrow \dfrac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \dfrac{4}{-2} = -2$

$\Rightarrow \dfrac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha} = -2$

$\Rightarrow \dfrac{\tan(\theta+\alpha)}{\tan\alpha} = -2$

$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$

16. R = {(x, y) : x and y were born in Delhi on same day}
R is an equivalence relation.

17. Required no. of subsects
$= {}^{10}C_2 + {}^{10}C_3 = 45 + 120 = 165$

18. $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3} = 0$

19. $(\alpha-\beta)^2 < 5$

$\Rightarrow k^2 - 4 < 5 \Rightarrow k^2 < 9$

$\Rightarrow -3 < k < 3$ ..... (1)

Next, $|k| \geq 2 \Rightarrow k \leq -2$ or $k \geq 2$ ..... (2)

from both, $k \in (-3, -2] \cup [2, 3)$

20. Let $\alpha, k\alpha$ be roots of $x^2 + px + q = 0$

and $\beta, k\beta$ be roots of $x^2 + \ell x + m = 0$

Clearly, $\dfrac{\alpha^2}{\beta^2} = \dfrac{p^2}{\ell^2} = \dfrac{q}{m} \Rightarrow p^2 m = \ell^2 q$

21. $\left(\dfrac{-1+i\sqrt{3}}{2}\right)^n + \left(\dfrac{-1-i\sqrt{3}}{2}\right)^n = \omega^n + \omega^{2n} = -1$, n is not multiple of 3

22. Sum $= 12\,(10^0 + 10^1 + 10^2)$
$= 111 \times 12 = 1332$

23. $0.3 + 0.33 + 0.333 + \ldots$

$= \dfrac{3}{10} + \dfrac{33}{100} + \dfrac{333}{1000} + \ldots$

$= \dfrac{3}{9}\left[\dfrac{9}{10} + \dfrac{99}{100} + \dfrac{999}{1000} + \ldots\right]$

$= \dfrac{1}{3}\left[\left(1-\dfrac{1}{10}\right) + \left(1-\dfrac{1}{100}\right) + \left(1-\dfrac{1}{1000}\right) + \ldots\right]$

$= \dfrac{1}{3}\left[n - \dfrac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{\frac{9}{10}}\right] = \dfrac{1}{3}\left[n - \dfrac{1}{9}\left(1-\dfrac{1}{10^n}\right)\right]$

24. $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2) = 0$

25. In an A.P
If $S_m = n$ and $S_n = m$ then $S_{m+n} = -(m+n)$

26. $\dfrac{1+2i}{1-(1-i)^2} = \dfrac{1+2i}{1-(-2i)} = \dfrac{1+2i}{1+2i} = 1$

Modulus = 1, Argument = 0

27. If graph of quadratic lies entirely above x-axis then $D > 0$. So both roots are complex.

28. $|z+1| = |z+4-3|$
$\leq |z+4| + |-3|$
maximum value of $|z+1| = 6$

29. $z^2 = 2\bar{z}$
Let $z = x + iy$
Now $x^2 - y^2 + 2xyi = 2x - 2yi$

$\Rightarrow x = -1$ and $y = \pm\sqrt{3}$

So, $z = -1 + \sqrt{3}i$ and $z = -1 - \sqrt{3}i$
are two roots

30. $\cot(\alpha+\beta) = \dfrac{\cot\alpha - \cot\beta - 1}{\cot\beta + \cot\alpha}$

$= \dfrac{c-1}{-b} = \dfrac{1-c}{b}$

31. $\tan 18° = \dfrac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ (Result)

32. $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$\Rightarrow \tan^{-1}\dfrac{2y}{1-y^2} = \tan^{-1}\dfrac{x+z}{1-xz}$

$\Rightarrow \dfrac{2y}{1-y^2} = \dfrac{x+z}{1-xz} = \dfrac{x+z}{1-y^2}$ $(\because y^2 = xz)$

$\Rightarrow 2y = x + z$

$\Rightarrow x, y, z \in$ A.P

But $x, y, z \in$ G.P (given)

$\Rightarrow x = y = z$

33. $\tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$

$= \dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \dfrac{2+1}{1-2\times 1} = \dfrac{3}{-1} = -3$

34. $A + B + C = \pi$

$\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \cos\frac{A}{2}$ (True)

$\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{A}{2}\right) = \cot\frac{A}{2}$ (True)

Statement 1 and 2 are true.

35. $\text{Sec}\,\theta - \text{cosec}\,\theta = \frac{4}{3}$

$\Rightarrow \frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = \frac{4}{3}$

$\Rightarrow \frac{(\sin\theta - \cos\theta)^2}{(\sin\theta\cos\theta)^2} = \frac{16}{9}$

$\Rightarrow \frac{1 - 2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta} = \frac{16}{9}$

Let $\sin\theta\ \cos\theta = x$

$\Rightarrow \frac{1-2x}{x^2} = \frac{16}{9}$

$\Rightarrow 16x^2 + 18x - 9 = 0$

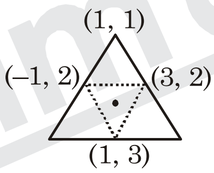
$\Rightarrow (8x - 3)(2x + 3) = 0$

$\Rightarrow x = \frac{3}{8}, x = \frac{-3}{2}$

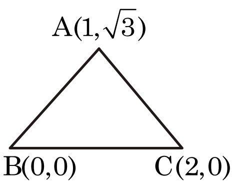
$\Rightarrow \sin\theta\ \cos\theta = \frac{3}{8}$

$\Rightarrow \sin\theta - \cos\theta = \frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$

36. Centroid $= \left(1, \frac{7}{3}\right)$

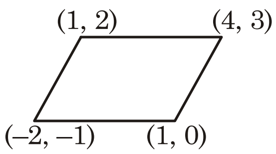


37. ΔABC is equilateral



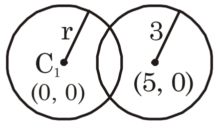
$\Rightarrow$ Incentre = Centroid $= \left(1, \frac{1}{\sqrt{3}}\right)$

38.



4^{th} vertex $= (4 - 2 - 1, 3 - 1 - 0) = (1, 2)$

39.



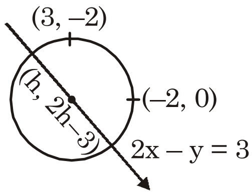
$r - 3 < C_1C_2 < r + 3$

$\Rightarrow r - 3 < 5 < r + 3$

$\Rightarrow r > 2, r < 8$

$\Rightarrow 2 < r < 8$

40. $(h - 3)^2 + (2h - 1)^2 = (h + 2)^2 + (2h - 3)^2$



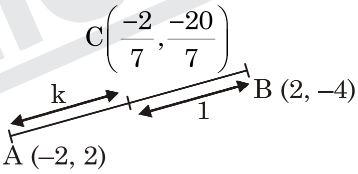
$\Rightarrow h = -\frac{3}{2} \Rightarrow 2h - 3 = -6$

$\therefore\ r^2 = \frac{1}{4} + 36$

Equation of circle is $\left(x + \frac{3}{2}\right)^2 + (y + 6)^2 = \frac{1}{4} + 36$

$\Rightarrow x^2 + y^2 + 3x + 12y + 2 = 0$

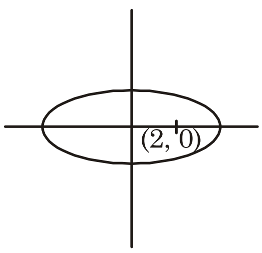
41. $\frac{2k-2}{k+1} = -\frac{2}{7}$



$\Rightarrow 7(k - 1) = -k - 1$

$\Rightarrow 8k = 6 \Rightarrow k = \frac{3}{4}$ ratio = 3 : 4

42. $2 = \frac{a}{4} \Rightarrow a = 8$



$b^2 = 64 - 4 = 60$

Equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{60} = 1$

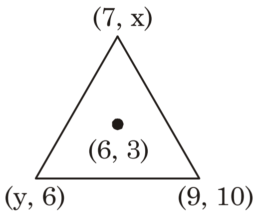
43. Let equation of line is $2x + 3y + \lambda = 0$

putting $(-1, 2)$, we get $\lambda = -4$

$\Rightarrow$ equation of line is $2x + 3y - 4 = 0$

44. $\theta = \tan^{-1}\left(\dfrac{\dfrac{-\sqrt{2}}{\sqrt{3}} + \dfrac{\sqrt{3}}{\sqrt{2}}}{1 + \left(\dfrac{-\sqrt{2}}{\sqrt{3}}\right)\left(\dfrac{-\sqrt{3}}{\sqrt{2}}\right)}\right) = \tan^{-1}\left(\dfrac{1}{2\sqrt{6}}\right)$

45.



$16 + y = 18 \Rightarrow y = 2$
Next, $x + 4 = 9 \Rightarrow x = 5$
$\therefore (x, y) = (5, 2)$

46. $-b = \dfrac{b^2 - 2c}{c^2}$

$\Rightarrow -bc^2 = b^2 - 2c \Rightarrow 2c = b^2 + bc^2$

$\Rightarrow \dfrac{2c}{b} = b + c^2 \Rightarrow \dfrac{2}{b} = \dfrac{b}{c} + c$

$\Rightarrow c, \dfrac{1}{b}, \dfrac{b}{c} \in$ A. P $\Rightarrow \dfrac{1}{c}, b, \dfrac{c}{b} \in$ H. P

47. $-\dfrac{1}{a} = \dfrac{\dfrac{1}{a^2} - \dfrac{2c}{a}}{\dfrac{c^2}{a^2}}$

$\Rightarrow -\dfrac{1}{a} = \dfrac{1 - 2ca}{c^2} \Rightarrow a - 2ca^2 = -c^2$

$\Rightarrow 2ca^2 = a + c^2 \Rightarrow a, ca^2, c^2 \in$ A. P

48. ${}^8C_1 + {}^8C_2 + \ldots\ldots\ldots {}^8C_7$
$= 2^8 - 2 = 254$

49. ${}^3P_2 \times 6! = 6 \times 720 = 4320$

50. $a_n = 2n - 3 \Rightarrow a_5 = 7$

51. $11 + 13 + \ldots\ldots + 99$

$= \dfrac{45}{2}(11 + 99) = 45 \times 55 = 2475$

52. $\dfrac{1}{2} + \dfrac{3}{4} + \dfrac{7}{8} + \dfrac{15}{16} + \ldots\ldots..$

$= \left(1 - \dfrac{1}{2}\right) + \left(1 - \dfrac{1}{4}\right) + \left(1 - \dfrac{1}{8}\right) + \ldots\ldots..$

$= n - \left(\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots\ldots.\right)$

$= n - \dfrac{\dfrac{1}{2}\left(1 - \dfrac{1}{2^n}\right)}{\dfrac{1}{2}} = n - 1 + 2^{-n}$

53. By property, $(A - B) \cup A = A$, $(A - B) \cap B = \phi$ and $A \subseteq B \Rightarrow A \cup B = B$ are true.

54. $(1p101)^2 + (10q1)_2 = (100\,r00)_2$
Equating, P = 0, q = 1, r = 0

55. $S = \{x : x^2 + 1 = 0, x \in R\}$
$\Rightarrow S = \phi$

56. ${}^nC_4\, x^{n-4} y^4 - {}^nC_5\, x^{n-5} y^5 = 0$

$\Rightarrow {}^nC_4\, x^{n-4} y^4 = {}^nC_5\, x^{n-5} y^5$

$\Rightarrow \dfrac{x}{y} = \dfrac{{}^nC_5}{{}^nC_4} = \dfrac{n-4}{5}$

57. $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} \alpha^2 + 4 & 4\alpha \\ 4\alpha & \alpha^2 + 4 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \alpha^3 + 12\alpha & 6\alpha^2 + 8 \\ 6\alpha^2 + 8 & \alpha^3 + 12\alpha \end{bmatrix}$

$\Rightarrow |A^3| = 125$

$\Rightarrow \left(\alpha^3 + 12\alpha\right)^2 - \left(6\alpha^2 + 8\right)^2 = 125$

putting $\alpha = \pm 3$, $(63)^2 - (62)^2 = 125$ satisfied.

58. $|B^{-1}AB| = |B^{-1}|\ |A|\ |B|$

$= \dfrac{1}{|B|}\ |A|\ |B| = |A|$

59. Putting x = 0

$\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$ is a skew-symmetric matrix of odd order

whose determinant value is zero.

60. $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$AA^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = Identity matrix

61. $\log\left(\dfrac{dy}{dx}\right) = a \Rightarrow \dfrac{dy}{dx} = e^a \Rightarrow \int dy = \int e^a\, dx$

$\Rightarrow y = x\, e^a + c$

62. $f(x) = \begin{cases} 2x + 1, -3 < x < -2 \\ x - 1, -2 \le x < 0 \\ x + 2, 0 \le x < 1 \end{cases}$

Here $f(0^-) = -1$ and $f(0^+) = 2$ clearly f(x) is discontinuous at x = 0 and continuous at all other points

63. If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exists, then $\lim_{x\to a} f(x).g(x)$ exists. But if $\lim_{x\to a} f(x).g(x)$ exists, then it is not necessery that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exists.

64. If $f(x) = x^2 (x-3)$ then $f(-x) = x^2(-x-3) \neq f(x)$ or $-f(x)$
$\Rightarrow$ f(x) is neither even nor odd.

65. $\frac{d}{dx} \log_{10}^{(5x^2+3)}$

$= \frac{1}{5x^2+3} \times \log_{10}^{e} \times 10x = \frac{10x \log_{10}^{e}}{5x^2+3}$

66. $f(a) = \frac{a-1}{a+1}$

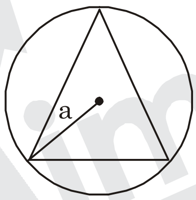
$f(2a) = \frac{2a-1}{2a+1}$

$f(a) + 1 = \frac{a-1}{a+1} + 1 = \frac{2a}{a+1}$

So, $f(2a) \neq f(a) + 1$

Next, $f\left(\frac{1}{a}\right) = \frac{\frac{1}{a}-1}{\frac{1}{a}+1} = \frac{1-a}{a+1} = -\left(\frac{a-1}{a+1}\right) = -f(a)$

67. For area of Δ to be maximum, triangle should be an equilateral triangle.



ℓ = length of side of equilateral triangle = $\sqrt{3}a$

$\therefore$ Area = $\frac{\sqrt{3}}{4}\left(\sqrt{3}\,a\right)^2 = \frac{3\sqrt{3}}{4}a^2$

68. $f(x) = x + \frac{1}{x} \Rightarrow f'(x) = \frac{x^2-1}{x^2}$

$\Rightarrow f'(x) = \frac{(x-1)(x+1)}{x^2}$

for $x \in (0, 1)$, $f'(x) < 0$
$\Rightarrow$ f(x) decreases

69. $f(x) = x^n$, $n \neq 0$.
$\Rightarrow f'(x) = n\,x^{n-1}$
f(x) to be differentiable, $n - 1 \geq 0 \Rightarrow n \geq 1$
$\Rightarrow n \in [1, \infty]$

70. $\int_{e^{-1}}^{e^2} \left|\frac{\log x}{x}\right| dx = \int_{e^{-1}}^{e^0} -\frac{\log x}{x}\, dx + \int_{e^0}^{e^2} \frac{\log x}{x}\, dx$

$= -\frac{1}{2}\left[(\log x)^2\right]_{e^{-1}}^{e^0} + \frac{1}{2}\left[(\log x)^2\right]_{e^0}^{e^2} = \frac{1}{2} + 2 = \frac{5}{2}$

71. New variance $= 5 \times (3)^2 = 45$

72. Required mean

$= \frac{20\times100-(21+21+18+20)}{96} = \frac{1920}{96} = 20$

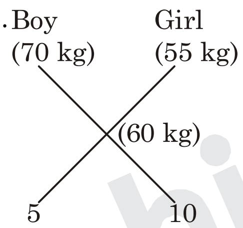
73. Required probability

$= \frac{{}^2C_0 \times {}^2C_2}{{}^4C_2} = \frac{1}{6}$

74. Required probability

$= 1 - \left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

75.



By aligation, ratio = 1 : 2

$\therefore$ no. of boys $= \frac{1}{3} \times 150 = 50$

76. Parallel to y-axis.

77. Let O (0, 0, 0) A (a, 0, 0), B (0, b, 0) and C (0, 0, c).

$P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from O, A, B and C.

78. P, Q, R, S are collinear.

79. Let $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{3} = \lambda$

$\Rightarrow x = 2\lambda + 1,\ y = -3\lambda + 2,\ z = 3\lambda - 1$

putting $x = 0,\ \lambda = -\frac{1}{2}$

$\therefore\ y = \frac{7}{2},\ z = -\frac{5}{2}$

So, point $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

80. $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$ ...(i)

$\frac{x-f}{e} = \frac{y-0}{1} = \frac{z-h}{g}$ ...(ii)

(i) and (ii) are perpendicular
$\Rightarrow ae + 1 + cg = 0$

81. $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$

$\Rightarrow 3n - 2m + 2n - 2 + 2m - 3 = 0$

$\Rightarrow n = 1$

Next, $1 + m^2 + n^2 = 6 \Rightarrow m = \pm 2$

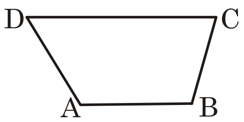
82. $\overrightarrow{OA} + \overrightarrow{OC} = 2\left(\overrightarrow{OP}\right)$ .... (1)

$\overrightarrow{OB} + \overrightarrow{OD} = 2\left(\overrightarrow{OP}\right)$ .... (2)

adding (1) and (2) we get,

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$

83.



$\overrightarrow{BA} + \overrightarrow{AD} = \overrightarrow{BD}$ ...(i)

$\overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{CA}$ ...(ii)

adding (i) and (ii)

$\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} + \overrightarrow{CA}$

84. $\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{b} \times \vec{c} = \vec{a} \Rightarrow \vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ form an orthogonal system.

Next, $|\vec{a} \times \vec{b}| = |\vec{c}| \Rightarrow |\vec{a}|\,|\vec{b}| \sin 90° \,.\, 1 = |\vec{c}|$

and $|\vec{b} \times \vec{c}| = \vec{a} \Rightarrow |\vec{b}|\,|\vec{c}| = \sin 90° \,.\, 1 = |\vec{a}|$

putting for $|\vec{c}|$, we get $|\vec{b}||\vec{a}||\vec{b}| = |\vec{a}| \Rightarrow |\vec{b}| = 1$

and $|\vec{a}| = |\vec{c}|$

85. (2) (3) + (3) (2) – $4\lambda = 0 \quad \Rightarrow \lambda = 3$

86. $\lim\limits_{x \to 0} \dfrac{e^x - (1+x)}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...) - (1+x)}{x^2} = \dfrac{1}{2!} = \dfrac{1}{2}$

87. $\displaystyle\int_0^{\pi/2} \frac{1}{1+\cos\theta} d\theta$

$= \displaystyle\int_0^{\pi/2} \frac{1}{2\cos^2\frac{\theta}{2}} d\theta = \frac{1}{2}\int_0^{\pi/2} \sec^2\frac{\theta}{2} d\theta$

$= \dfrac{1}{2} \times 2\left[\tan\dfrac{\theta}{2}\right]_0^{\pi/2} = 1$

88. $\displaystyle\int \frac{1}{x(x^7+1)} dx$

$= \dfrac{1}{7}\displaystyle\int \frac{7x^6}{x^7(x^7+1)} dx$

putting $x^7 = t$

$= \dfrac{1}{7}\displaystyle\int \frac{7x^6}{t(t+1)} dt = \frac{1}{7}\int\left(\frac{1}{t} - \frac{1}{t+1}\right) dt$

$= \dfrac{1}{7}\,[\log t - \log(t+1)] + c$

$= \dfrac{1}{7}\log\left(\dfrac{t}{t+1}\right) + c = \dfrac{1}{7}\log\left(\dfrac{x^7}{x^7+1}\right) + c$

89. $f(x) = \cos x$ is bijective for domain $X = [0, \pi]$ and co-domain $Y = [-1, 1]$.

90. $\dfrac{f(a)}{f(a+1)} = \dfrac{\frac{a}{a-1}}{\frac{a+1}{a}} = \dfrac{a}{a-1} \times \dfrac{a}{a+1} = \dfrac{a^2}{a^2-1} = f(a^2)$

91. $A \subseteq B \Rightarrow A \cap B = A$

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)} = \dfrac{0.2}{0.5} = \dfrac{2}{5}$

$P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(A)}{P(A)} = 1$

92. Required probability

$= \dfrac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \dfrac{1}{4}$

93. $r = \sqrt{\dfrac{15}{4} \times \dfrac{2}{30}} = \dfrac{1}{2}$

94. Angle $= \dfrac{2}{5} \times 360° = 144°$

95. It is a fundamental concept. So, options 'a' is correct.

96. P(ace) $= \dfrac{4}{52} = \dfrac{1}{13}$

97. S.D $= \dfrac{5}{4}$ M.D. (formula)

98. Data can be represented in tabular and graphical form.

99. The abscissa of the point of intersection of less than and more than ogive is median.

100. Both statements are correct

101.Result = $\sqrt{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{8}\right)} = -\frac{1}{4}$

102.Median remains same but the mean will decrease.

103.Required probability

$= 1 - \frac{9}{36} = 1 - \frac{1}{4} = \frac{3}{4}$

104.P $(\overline{A}\cap\overline{B}) = 1 - P(A\cup B)$

$= 1 - \left(\frac{1}{3}+\frac{1}{4}\right) = \frac{5}{12}$

105.np = 12 and npq = 4

$\Rightarrow q = \frac{4}{12} = \frac{1}{3} \Rightarrow p = \frac{2}{3}$

So, $n \times \frac{2}{3} = 12 \Rightarrow n = 18$

106. $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$

$= \frac{1}{e}\int \frac{ex^{e-1}+e^x}{x^e+e^x}dx$

$= \frac{1}{e}\log(x^e+e^x)+c$

107.$f(x) = x^2 - 3$

f o f (x) = $(x^2-3)^2 - 3 = x^4 - 6x^2 + 6$

f o f o f (x) = $(x^4 - 6x^2 + 6)^2 - 3$

f o f o f (x) is even function

$\Rightarrow$ (f o f o f) (–1) = (f o f o f) (1)

Next (f o f o f) (–1) –4 (f o f o f) (1)

–3 {(f o f o f) (1)} = (–3) (–2) = 6

f o f (o) = 6

Both 1 and 2 are correct.

108.p(mx + n) + q = m(px + q) + n

$\Rightarrow$ pn + q = qm + n

$\Rightarrow$ f(n) = g(q)

109. $\mu m_{x\to 1} \frac{F(x)-F(1)}{x-1}$

$= \{F'(x)\}_{x=1} = \left(\frac{-2x}{2\sqrt{9-x^2}}\right)_{x=1}$

$= \frac{-1}{2\sqrt{2}}$

110. $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{1}{\left(\frac{dy}{dx}\right)}\right)$

$= \frac{-\frac{d}{dy}\left(\frac{dy}{dx}\right)}{\left(\frac{dy}{dx}\right)^2} = \frac{-\frac{d}{dx}\left(\frac{dy}{dx}\right)\times\frac{dx}{dy}}{\left(\frac{dy}{dx}\right)^2}$

$= -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

111.$(f-g)(x) = \begin{cases} x, & x\in Q \\ -x, & x\in R-Q \end{cases}$

clearly $f-g$ is one-one and on to

112.f(x) = sin 3x

sin 3x is increasing in

$\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

Interval length = $\frac{\pi}{3}$

113.$x\,dy = ydx + y^2dy$

$\Rightarrow \int\frac{ydx - xdy}{y^2} = -\int dy \Rightarrow -y = \frac{x}{y} + C$

$y(1) = 1 \Rightarrow C = -2 \Rightarrow -y = \frac{x}{y} - 2$

$\therefore y(-3) = 3\ (\because y(x) > 0)$

114.Maximum value = 4 + 1 = 5

115.$f(x) = \int \sin^2 x dx = \int\left(\frac{1-\cos 2x}{2}\right)dx$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$

$f(x+\pi) \neq f(x)$ ($\because$ f(x) is not periodic)

Statement 1 false

Next, $\sin^2(\pi + x) = \sin^2 x$

Statement 2 true.

116.$y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^{-2}$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 = x\left(\frac{dy}{dx}\right)^4 + 1$

order = 1, degree = 4

117. $y^2 - 2ay + x^2 = a^2$

$$\Rightarrow 2y\frac{dy}{dx} - 2a\frac{dy}{dx} + 2x = 0 \Rightarrow \frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}} = a$$

$$\Rightarrow \frac{py + x}{p} = a \Rightarrow y^2 \frac{-2y(py+x)}{p} + x^2 = \left(\frac{py+x}{p}\right)^2$$

$\Rightarrow p^2y^2 - 2p^2y^2 - 2xyp + p^2x^2 = p^2y^2 + x^2 + 2xyp$

$\Rightarrow -2p^2y^2 + p^2x^2 - 4xyp - x^2 = 0$

$\Rightarrow p^2(x^2 - 2y^2) - 4xyp - x^2 = 0$

118. $ydx = (x + 2y^2)dy$

$$\Rightarrow \frac{dx}{xy} - \frac{x}{y} - 2y$$

$$\text{I.F} = e^{\int -\frac{1}{y}dy} = -\frac{1}{y}$$

Solution is given by, $-\frac{x}{y} = \int -2dy$

$$\Rightarrow \frac{x}{y} = 2y + c \quad \Rightarrow x = 2y^2 + cy$$

119. $f(x + y) = f(x).\,f(y)\ \forall\ x,y \in R$

$\Rightarrow f(0 + 0) = f(0).f(0) \Rightarrow \{f(0)\}^2 - f(0) = 0 \Rightarrow f(0) = 1$

Next, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{f(x).f(h) - f(x)}{h} = f(x) \cdot \lim_{h\to 0} \frac{f(h) - 1}{h}$$

$$= f(x) \lim_{h\to 0} \frac{f(h) - f(o)}{h} = f(x).\,f'(o)$$

$\therefore f'(5) = f(5)\,.\,f'(o).$

120. $I = \int_0^a f(x)\,g(x)dx$

$$= \int_0^a f(a-x).g(a-x)dx = \int_0^a f(x)\{2 - g(x)\}dx$$

$$= \int_0^a 2f(x)\,dx - \int_0^a f(x).g(x)\,dx$$

$$\Rightarrow 2I = 2\int_0^a f(x)dx \Rightarrow I = \int_0^a f(x)dx$$