NDA II 2018 Maths Solutions Code-A

Date: 9 September 2018

1. (C)

$\sqrt{7\sqrt{7\sqrt{7}}} = 7^{1/2} \cdot 7^{1/4} \cdot 7^{1/8} = 7^{7/8}$

$\log_7 \log_7 \left(7^{7/8}\right) = \log_7 (7/8) = \log_7 7 - \log_7 2^3$

$= 1 - 3\log_7 2$

2. (C)

$\frac{x}{1-r} = 5 \Rightarrow \frac{x}{5} = 1 - r \Rightarrow r = 1 - \frac{x}{5}$

where $|r| < 1$

$\Rightarrow \; -1 < 1 - \frac{x}{5} < 1$

$\Rightarrow \; -2 < -\frac{x}{5} < 0$

$\Rightarrow \; -10 < -x < 0$

$\Rightarrow \; 10 > x > 0$

3. (A)

By definition of rational function.
Option (A) is correct.

4. (B)

If $A^{-1} = A^T$, then A is orthogonal matrix.

5. (C)

Checking through option 'C" is incorrect.

6. (B)

The no. with all distinct digits = 5 × 9 × 8 × 7
= 2520

$\therefore$ x = 5001 – 2520 = 2481

7. (C)

$\frac{3}{1-\left(-\frac{1}{3}\right)} = \frac{9}{4}$

8. (A)

300 = 125 + 145 + 90 –

$\left(|A \cap B| + |B \cap C| + |A \cap C|\right) + |A \cap B \cap C|$

$|A \cap B| + |B \cap C| + |A \cap C| = 60 + |A \cap B \cap C|$ ...(i)

Again,

$|A \cap B| + |B \cap C| + |A \cap C| - 3|A \cap B \cap C| = 32$

$\Rightarrow \; |A \cap B| + |B \cap C| + |A \cap C| = 32 + 3|A \cap B \cap C|$ ...(ii)

From (i) and (ii)

$|A \cap B \cap C| = 14$

9. (C)

Exactly One = $|A| + |B| + |C| -$

$2\left[|A \cap B| + |B \cap C| + |A \cap C|\right] + 3|A \cap B \cap C|$

= 125 + 145 + 90 – 2[32 + 3 × 14] + 3 × 14

= 360 – 106 = 254

10. (D)

$\alpha + \beta = -\alpha, \; \alpha\beta = -\beta \Rightarrow \alpha\beta + \beta = 0$

$\Rightarrow \; (\alpha + 1)\beta = 0 \Rightarrow \alpha = -1 \quad (\beta \neq 0)$

$\Rightarrow \; 2\alpha + \beta = 0$

$\Rightarrow \; \beta = 2$

$\therefore \; -x^2 + \alpha x + \beta = -x^2 - x + 2$

Greatest value = $-\frac{1+8}{-4} = \frac{9}{4}$

11. (D)

$4c_2 \times 2^2 \times 3^2$

= 6 × 4 × 9 = 216

12. (A)

Statement 1 and 2 are correct.
Statement 3 is incorrect because

$(\lambda A)^{-1} = \frac{1}{\lambda} A^{-1}, \lambda \neq 0$

13. (A)

$C_1 \to C_1 - C_3$

$$\begin{vmatrix} x-3 & y & 3 \\ x^2-9 & 5y^3 & 9 \\ x^3-27 & 10y^5 & 27 \end{vmatrix}$$

$$(x-3)\begin{vmatrix} 1 & y & 3 \\ x+3 & 5y^3 & 9 \\ x^2+9-34 & 10y^5 & 27 \end{vmatrix}$$

14. (A)

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\text{adj } A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

15. (B)

$(w)^{3n} + \left(w^2\right)^{3n}$

$\left(w^3\right)^n + \left(w^3\right)^{2n}$

= 1 + 1 = 2

16. (D)

$= {}^{12}C_8 \times {}^{5}C_3$

17. (B)

$$\frac{3}{2}+\frac{5}{3}=\frac{19}{6}$$

18. (A)

$(AB)^{-1} = B^{-1}A^{-1}$

19. (D)

Going through option, $x = 0$

20. (B)

$$\begin{vmatrix} 2 & 4 \\ -8 & x \end{vmatrix} = 0 \Rightarrow x = -16$$

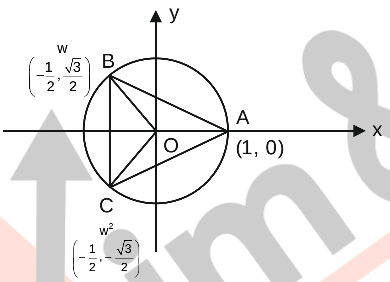
21. (B)

$$\text{Since } \begin{vmatrix} 2 & 1 & -3 \\ 3 & -2 & 2 \\ 5 & -3 & -1 \end{vmatrix} = 26 \neq 0$$

$\Rightarrow$ System is consistent with unique solution.

22. (C)

Δ is equilateral.



23. (A)

$$\begin{vmatrix} \log u & p & 1 \\ \log v & q & 1 \\ \log w & r & 1 \end{vmatrix} = 0$$

($\because$ u, v, w are in G.P. $\Rightarrow$ log u, log v, log w are in A.P)

24. (C)

$\alpha = {}^{2n}C_n$

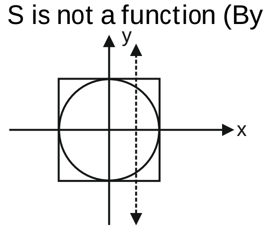
$\beta = {}^{2n-1}C_n$

$\gamma = {}^{2n-1}C_{n-1}$

$\beta + \gamma = {}^{2n-1}C_n + {}^{2n-1}C_{n-1} = {}^{2n}C_n = \alpha$

25. (D)

S is not a function (By vertical line Test)



26. (C)

$T_n = \frac{1}{m}$, $T_m = \frac{1}{n}$

$\Rightarrow$ 1st Term = c.d. = $\frac{1}{mn}$

$\Rightarrow T_{mn} = \frac{1}{mn} + \frac{mn-1}{mn} = 1$

27. (B)

Let $f(x) = ax^2 + bx + c$, $a > 0$, $b^2 < 4ac$

($\because f(x) > 0$)

Now, $g(x) = ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (b+2a)x + 2a + b + c$

Now, $(b+2a)^2 - 4a(2a+b+c)$

$= b^2 + 4ab + 4a^2 - 8a^2 - 4ab - 4ac$

$= b^2 - 4ac - 8a^2 < 0 \Rightarrow g(x) > 0$

28. (C)

By property, statement 1 and 3 are correct.

29. (B)

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 101101101
−10110110
 10110111
  −11011
 10011100
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$\Rightarrow x = 1, y = 0$

30. (B)

B = adj A, ℓ = Identity matrix, |A| = k

AB = A(adj A) = |A| ℓ = k ℓ

31. (C)

$(0.2)^x = 2$

$x \log_{10} 2/10 = \log_{10} 2$

$x[\log_{10} 2 - \log_{10} 10] = \log_{10} 2$

$x[0.3010 - 1] = 0.3010$

$x = -\frac{0.3010}{0.6990} \approx -0.43$

32. (C)

$9 \times 9 \times 8 \times 7 \times 6$

$= 27,216$

33. (D)

$$\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{vmatrix} = R_1 \to R_1 + R_2 + R_3$$

$$\begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$= (x+y+z)(z-x)^2$, (replacing z by x)

44. (B)

$R_1 \to R_1 - R_2, R_3 \to R_3 - R_2$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin^2 A - 1 & \sin^2 B - 1 & \sin^2 C - 1 \end{vmatrix} = 0$$

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1 & 1 & 1 \\ -\cos^2 A & -\cos^2 B & -\cos^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0 \quad (R_3 \to R_3 + R_2)$$

$\Rightarrow \sin A = \sin B = \sin C \Rightarrow \Delta$ is equilateral.

35. (B)

Matrix product is commutative if both are diagonal matrices of same order.

$\Rightarrow A^2 - B^2 = (A+B)(A-B)$ is not true.

Next, $(A-I)(A+I) = 0$

$\Rightarrow A^2 + AI - IA - I^2 = 0$ ($\because$ AI = IA)

$\Rightarrow A^2 = I^2$ is correct.

36. (C)

$$\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta \text{ (formula)}$$

37. (A)

$$\frac{2}{\cos\theta} = \frac{\cos(\theta+\alpha)+\cos(\theta-\alpha)}{\cos(\theta+\alpha)\cos(\theta-\alpha)} = \frac{2\cos\theta\cdot\cos\alpha}{\cos^2\theta - \sin^2\alpha}$$

$\Rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$

$\Rightarrow \sin^2\alpha = \cos^2\theta(1-\cos\alpha)$

$\Rightarrow \cos^2\theta = \dfrac{\sin^2\alpha}{1-\cos\alpha} = 1+\cos\alpha$

$\Rightarrow 1 - \sin^2\theta = 1 + \cos\alpha$

$\Rightarrow \sin^2\theta + \cos\alpha = 0$

38. (D)

$\sin 2A - \sin 2B - \sin 2C$

$= 2\cos(A+B)\cdot\sin(A-B) + \sin(2A+2B)$

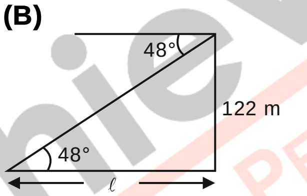
$= 2\cos(A+B)\sin(A-B) +$
$2\sin(A+B)\cos(A+B)$

$= 2\cos(A+B)[\sin(A-B)+\sin(A+B)]$

$= -2\cos C[2\sin A\cdot\cos B]$

$= -4\sin A\cos B\cos C$

39. (B)



$\ell = \dfrac{122}{\tan 48°} = 122\tan 42°$

40. (A)

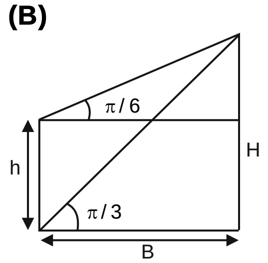
Checking through options

$300° = -60°$

So, $3\left[3 - \tan^2(-60°) - \cot(-60°)\right]^2$

$= 3\left[3 - 3 + \dfrac{1}{\sqrt{3}}\right]^2 = 3 \times \dfrac{1}{3} = 1$

41. (B)



$\beta = \dfrac{H}{\tan\frac{\pi}{3}} = \dfrac{H-h}{\tan\frac{\pi}{6}}$

$\Rightarrow \dfrac{H}{3} = H - h \Rightarrow \dfrac{2}{3}H = h \Rightarrow H = \dfrac{3}{2}h$

42. (B)

$\operatorname{cosec} x + \cot x = \sqrt{3}$

$\operatorname{cosec} x - \cot x = \frac{1}{\sqrt{3}}$

$\Rightarrow \; 2\operatorname{cosec} x = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow \; \operatorname{cosec} x = \frac{2}{\sqrt{3}} \Rightarrow \sin x = \frac{\sqrt{3}}{2}$

Possible values of x = $\pi/3, 2\pi/3$

$\therefore$ Option (B) is correct.

43. (B)

$\left[(2\cos\theta+1)(2\cos\theta-1)\right]^{10}\left[2\cos 2\theta-1\right]^{10}\left[2\cos 4\theta-1\right]^{10}$,

$\theta = \frac{\pi}{8}$

$= \left(4\cos^2\frac{\pi}{8}-1\right)^{10}\left(2\times\frac{1}{\sqrt{2}}-1\right)^{10}(-1)^{10}$

$= \left(4\times\frac{2+\sqrt{2}}{4}-1\right)^{10}\left(\sqrt{2}-1\right)^{10}$

$= \left[\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)\right]^{10} = 1^{10} = 1$

44. (A)

$\cos\alpha\cdot\cos\beta = -\frac{3}{4}$

$\Rightarrow \; \frac{1}{\cos\alpha\cdot\cos\beta} = \sec\alpha\cdot\sec\beta = -\frac{4}{3}$

45. (A)

$\tan^{-1}\left(\frac{2x+3x}{1-2x\cdot 3x}\right) = \frac{\pi}{4}$

$\tan^{-1}\frac{5x}{1-6x^2} = \frac{\pi}{4}$

$\frac{5x}{1-6x^2} = 1$

$1-6x^2 = 5x$

$6x^2+5x-1 = 0$

$(6x-1)(x+1) = 0$

$x = \frac{1}{6}, -1 \quad (\text{Rejected})$

$\Rightarrow x = \frac{1}{6}$

46. (C)

$ar = 2, \; \frac{a}{1-r} = 8 \;\Rightarrow\; \frac{ar}{r-r^2} = 8$

$\Rightarrow \; 2 = 8r - 8r^2 \Rightarrow 4r^2 - 4r + 1 = 0 \Rightarrow r = \frac{1}{2}$

$\therefore$ G.P. is 4, 2, 1, $\frac{1}{2}$, ...

47. (C)

$a, b, c \in A.P \Rightarrow a-b = b-c \Rightarrow \frac{a-b}{b-c} = 1$

$a, b, c \in G.P \Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{a-b}{b-c}$

$a, b, c \in H.P \Rightarrow b = \frac{2ac}{a+c} \Rightarrow ab + bc = 2ac$

$\Rightarrow ab - ac = ac - bc$

$\Rightarrow \frac{a-b}{b-c} = \frac{a}{c}$

48. (A)

543 + 534 + 453 + 435 + 354 + 345 = 2664

49. (B)

$\frac{D_1}{D}$ = (ratio of coefficient of x)² = $\frac{b^2}{q^2}$

50. (C)

$A = \sin^2\theta + \cos^4\theta = \sin^2\theta + \left(1-\sin^2\theta\right)^2$

Let $\sin^2\theta = x$

$= x^2 - x + 1, \quad 0 \le x \le 1$

$= \left(x-\frac{1}{2}\right)^2 + \frac{3}{4}$

$A(0) = A(1) = 1$

So, $\frac{3}{4} \le A \le 1$

51. (D)

Equation of circle is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$

52. (A)

$\left(x^2-2x+1\right)+\left(4y^2-4y+1\right) = 0$

$\Rightarrow \; (x-1)^2 + (2y-1)^2 = 0$

$\Rightarrow \; x = 1, \; y = \frac{1}{2}$

It is a point.

53. (C)

$$\tan^{-1}(\theta)=\tan^{-1}\left|\frac{\ell m'-\ell' m}{\ell\ell'+mm'}\right| \Rightarrow \theta=\left|\frac{\ell m'-\ell' m}{\ell\ell'+mm'}\right|$$

54. (B)

Using distance between two parallel lines formula.

55. (A)

Point of intersection $\left(\frac{6}{5},\frac{6}{5}\right)$

Let equation of line be 4x + 5y + k = 0

Putting $\left(\frac{6}{5},\frac{6}{5}\right)$, $k=-\frac{54}{5}$

$\therefore$ Equation of line is 20x + 25y – 54 = 0

56. (A)

$$\text{Distance} = \left|\frac{3\times2-6\times3+2\times4+11}{\sqrt{3^2+(-6)^2+(2)^2}}\right|$$

$$=\left|\frac{6-18+8+11}{\sqrt{49}}\right| = 1$$

57. (C)

go through the option (C)

58. (B)

drs of line is 2, 3, 4

going through option,

$2(1)+3(2)+4(-2)=0$,

$2(4)+3(4)-4(5)=0$

59. (C)

Angle between planes

$$=\cos^{-1}\left(\frac{2-1+2}{6}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

Distance between planes

$$=\frac{\left|\frac{2}{3}-4\right|}{\sqrt{99}}=\frac{10}{3\times3}=\frac{10}{9}$$

60. (D)

$$\cos\alpha=\frac{mr+ns}{\sqrt{m^2+n^2}\ \sqrt{r^2+s^2}}$$

Statement 1 is false, statement 2 is true.

61. (D)

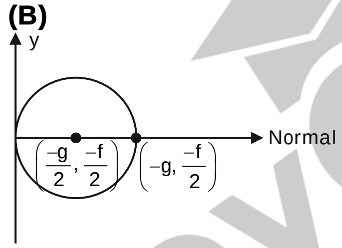
$$\text{Area}=\frac{1}{2}\left|x_1\left(\frac{1}{x_2}-\frac{1}{x_3}\right)+x_2\left(\frac{1}{x_3}-\frac{1}{x_1}\right)+x_3\left(\frac{1}{x_1}-\frac{1}{x_2}\right)\right|$$

$$=\frac{1}{2}\left|\frac{x_1(x_3-x_2)}{x_2x_3}+\frac{x_2(x_1-x_3)}{x_1x_3}+\frac{x_3(x_2-x_1)}{x_1x_2}\right|$$

$$=\frac{1}{2}\left|\frac{-x_1^2(x_2-x_3)-x_2^2(x_3-x_1)-x_3^2(x_1-x_2)}{x_1x_2x_3}\right|$$

$$=\left|\frac{(x_1-x_2)(x_2-x_3)(x_3-x_1)}{2(x_1x_2x_3)}\right|$$

62. (B)



63. (A)

$$(\vec a+\vec b)\cdot(\vec a+\vec b)=a^2+2\vec a\cdot\vec b+b^2=a^2+b^2$$

$$\Rightarrow\ \vec a\cdot\vec b=0\ \Rightarrow\ \vec a\perp\vec b$$

64. (D)

$$(x\hat i+y\hat j+z\hat k)\cdot(\hat i+\hat j+\hat k)$$

$= x + y + z$

65. (C)

$$\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & -4 & -1 \end{vmatrix}$$

$$=\hat i(1+4)-\hat j(-2-3)+\hat k(-8+3)$$

$$=5\hat i+5\hat j-5\hat k$$

$$\text{Unit vector}=\frac{5}{5\sqrt3}\hat i+\frac{5}{5\sqrt3}\hat j-\frac{5}{5\sqrt3}\hat k$$

$$=\frac{1}{\sqrt3}\hat i+\frac{1}{\sqrt3}\hat j-\frac{1}{\sqrt3}\hat k$$

66. (D)

$$|\vec a+\vec b|^2+|\vec a-\vec b|^2=2\{|\vec a|^2+|\vec b|^2\}=2\times25=50$$

$$\Rightarrow |\vec a+\vec b|^2+25=50 \Rightarrow |\vec a+\vec b|^2=25 \Rightarrow |\vec a+\vec b|=5$$

67. (C)

for simplicity let us take $\vec{a}, \vec{b}, \vec{c}$ as $\hat{i}, \hat{j}, \hat{k}$.

Now magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ will be $\sqrt{3}$.

68. (C)

$\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$

$= \vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}+\vec{b}-\vec{b}$

$= 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}+0$

$= 2\left(\vec{a}\times\vec{b}\right)$

69. (C)

$\vec{\tau}=\vec{r}\times\vec{F}$

$=\left(\hat{i}+2\hat{j}+3\hat{k}\right)\times\lambda\hat{k}$

$=2\lambda\hat{i}-\lambda\hat{j}$

$\Rightarrow\ \left|\vec{\tau}\right|=\sqrt{5}\ \lambda$

70. (A)

From Δ law of vector addition

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

Only statement (1) is correct.

71. (B)

$y=\cos^{-1}\left(\sin x\right)=\cos^{-1}\cos\left(\frac{\pi}{2}-x\right)=\frac{\pi}{2}-x$

$\tan\theta=-1 \quad \Rightarrow\ \theta=\frac{3\pi}{4}$

72. (D)

f(x) is defined if $x\geq 1$ and $x\neq 4$

$\therefore$ Domain = $\left[1,4\right)\cup\left(4,\infty\right]$

73. (A)

$\lim\limits_{x\to 0}\frac{\sin 2x}{5x}=\frac{2}{5}\neq\frac{2}{15}$

$\Rightarrow$ f(x) is not continuous at x = 0

74. (A)

f(x) = |x – 3| is continuous everywhere and not diff. at x = 3

Option ‘a’ is incorrect.

75. (B)

$$f\left(0\right)=\lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}=\lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}=\frac{2-1}{2+1}=\frac{1}{3}$$

76. (A)

$$\lim_{x\to 1}\frac{f\left(x\right)-f\left(1\right)}{x-1}=f'\left(1\right)=\frac{-1}{\sqrt{24}}$$

$$\because\ f'\left(x\right)=\frac{-2x}{2\sqrt{25-x^2}}=\frac{-x}{\sqrt{25-x^2}}$$

77. (A)

$$y=\tan^{-1}\left(\frac{5-2\tan\sqrt{x}}{2+5\tan\sqrt{x}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{5}{2}-\tan\sqrt{x}}{1+\left(\frac{5}{2}\right)\tan\sqrt{x}}\right)$$

$$=\tan^{-1}\frac{5}{2}-\tan^{-1}\tan\sqrt{x}=\tan^{-1}\frac{5}{2}-\sqrt{x}$$

$$\therefore\ \frac{dy}{dx}=-\frac{1}{2\sqrt{x}}$$

78. (A)

$f\left(x\right)=x\sin x+\cos x+\frac{1}{2}\cos^2 x$

$\Rightarrow\ f'\left(x\right)=x\cos x+\sin x-\sin x-\sin x\cos x$

$=\cos x\left(x-\sin x\right)>0$ in $\left(0,\frac{\pi}{2}\right)$

79. (C)

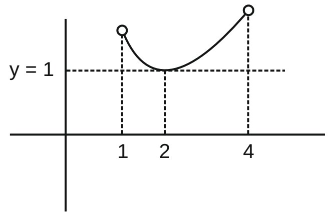
$$\lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta}=\lim_{\theta\to 0}\frac{\sqrt{2}\sin\left(\theta/2\right)}{\theta}$$

$$=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$

80. (C)

$f\left(x\right)=x^2-4x+5=\left(x-2\right)^2+1$

$f\left(1\right)=2,\ f\left(4\right)=5$



$\therefore$ Range f(x) = [1, 5)

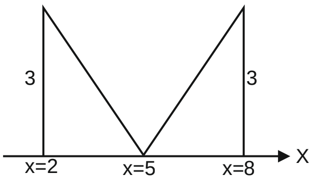
81. (B)

$$\int_a^b\left[x\right]dx+\int_a^b\left[-x\right]dx$$

$$=\int_a^b\left(\left[x\right]+\left[-x\right]\right)dx\ =\int_a^b\left(-1\right)dx=a-b$$

82. (D)

$$\int_2^8 |x-5|\,dx = 2\times\frac{1}{2}\times 3\times 3 = 9$$



83. (D)

Let $t = \sin x \Rightarrow dt = \cos x\,dx$

$$\int \sin^3 x\cos x\,dx = \int t^3 dt = \frac{t^4}{4}+c = \frac{\sin^4 x}{4}+c$$

$$= \frac{\left(1-\cos^2 x\right)^2}{4}+c$$

84. (B)

$$\int e^{\ell n(\tan x)}dx = \int \tan x\,dx$$

$$= \ell n\left|\sec x\right| + c$$

85. (C)

$$\int_{-1}^{1}\left(\frac{d}{dx}\tan^{-1}\frac{1}{x}\right)dx$$

$$= \int_{-1}^{1}\left(\frac{d}{dx}\cot^{-1}x\right)dx$$

$$= \int_{-1}^{1}\frac{-1}{\left(1+x^2\right)}dx = -2\int_0^1\frac{1}{1+x^2}dx$$

$$= -2\left[\tan^{-1}x\right]_0^1 = -2\times\frac{\pi}{4} = -\frac{\pi}{2}$$

86. (A)

$$f(x) = x^2-5x+6$$

$$\Rightarrow f'(x) = 2x-5$$

$$\Rightarrow f'(x) < 0$$

$$\Rightarrow 2x-5<0$$

$$\Rightarrow x < \frac{5}{2}$$

87. (D)

$y = p\cos ax + q\sin ax$

$$\Rightarrow \frac{dy}{dx} = -p\,a\sin ax + qa\cos ax$$

$$\Rightarrow \frac{d^2y}{dx^2} = -p\,a^2\cos ax - qa^2\sin ax = -a^2y$$

$$\Rightarrow \frac{d^2y}{dx^2}+a^2y = 0$$

88. (B)

$$\frac{dy}{dx} = -x^2-\frac{1}{x^3}$$

$$\Rightarrow \int dy = \int\left(-x^2-\frac{1}{x^3}\right)dx$$

$$\Rightarrow y = -\frac{x^3}{3}+\frac{1}{2x^2}+c$$

Putting $(-1,-2)$, we get

$$-2 = \frac{1}{3}+\frac{1}{2}+c \;\Rightarrow\; -\frac{17}{6}$$

$$\therefore\; y = -\frac{x^3}{3}+\frac{1}{2x^2}-\frac{17}{6}$$

$$\Rightarrow 6x^2y = -2x^5+3-17x^2$$

$$\Rightarrow 6x^2y+17x^2+2x^5-3 = 0$$

89. (D)

Order = 4 ($\because$ No. of arbitrary constants = 4)

90. (D)

$$\frac{dy}{dx} = e^{ax+by} = e^{ax}\cdot e^{by}$$

$$\Rightarrow e^{ax}dx - e^{-by}dy = 0$$

Integrating both sides,

$$\frac{1}{a}e^{ax}+\frac{1}{b}e^{-by} = c$$

91. (A)

$$u\frac{du}{dx}+v\frac{dv}{dx}$$

$$= e^{ax}\sin bx\left[ae^{ax}\sin bx + be^{ax}\cos bx\right]$$

$$+e^{ax}\cos bx\left[ae^{ax}\cos bx - be^{ax}\sin bx\right]$$

$$= e^{2ax}[a\sin^2 bx + b\sin bx\cos bx +$$

$$a\cos^2 bx - b\sin bx\cos bx]$$

$$= ae^{2ax}$$

92. (C)

$$y = \sin(\log x) \Rightarrow \frac{dy}{dx} = \frac{\cos(\log x)}{x}$$

$$\Rightarrow x\left(\frac{dy}{dx}\right) = \cos(\log x)$$

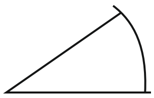
Again, Differentiating,

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-\frac{\sin\log x}{x}=\frac{-y}{x}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

93. (C)

$\ell+2r=40$



Area is maximum, when $\ell+2r=20 \;\Rightarrow\; r=10$

94. (A)

$\left[x(x-1)+1\right]^{1/3}$ is minimum when x^2-x+1 is minimum, $0\le x\le 1$

minimum value of $x^2-x+1=\frac{3}{4}$

$\Rightarrow$ Required value = $\left(\frac{3}{4}\right)^{1/3}$.

95. (A)

$$y=(-\sin x)^{-x}$$

$$\frac{dy}{dx}=(-\sin)^{-x}\left[\frac{x}{\sin x}(-\cos x)+\log(-\sin x)\cdot(-1)\right]$$

$$\left[\frac{dy}{dx}\right]x=-\frac{\pi}{6}\quad=\left(\frac{1}{2}\right)^{\pi/6}\left[-\frac{\pi}{6}\sqrt{3}-\log\frac{1}{2}\right]$$

$$2^{-\pi/6}\left(\frac{6\log 2-\sqrt{3}\,\pi}{6}\right)$$

96. (A)

For $\frac{\pi}{4}<x<\frac{\pi}{2}$, $\sqrt{1-\sin 2x}=\sin x-\cos x$

$\therefore$ Result = $\cos x+\sin x$

97. (A)

$$I=\frac{1}{a^2}\int\frac{\sec^2 x}{\tan^2 x+\left(\frac{b}{a}\right)^2}=\frac{1}{a^2}\times\frac{a}{b}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$$

$$=c+\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$$

98. (D)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)\left[f(h)-1\right]}{h}=\lim_{h\to 0}\frac{f(x)\left[h\,g(h)\phi(h)\right]}{h}$$

$$=ab\,f(x)$$

99. (C)

Let $t=x+y \;\Rightarrow\; \frac{dt}{dy}=\frac{dx}{dy}+1$

So, $\frac{dt}{dy}-1=\frac{t+1}{t-1}\Rightarrow\frac{dt}{dy}=\frac{t+1+t-1}{t-1}=\frac{2t}{t-1}$

$$\Rightarrow \int\frac{t-1}{t}dt=2\int dy$$

$$\Rightarrow t-\log t=2y+C_1$$

$$\Rightarrow x+y-\log(x+y)-2y=C_1$$

$$\Rightarrow y-x+\log(x+y)=-C_1=C$$

100. (D)

$$\lim_{x\to\frac{\pi}{6}}\frac{(\sin x+1)(2\sin x-1)}{(\sin x-1)(2\sin x-1)}=\frac{\frac{1}{2}+1}{\frac{1}{2}-1}=-3$$

101. (C)

n(S) = 36

A = {(1,5,) , (2,5), (3,5) (4,5) (5,5), (6,5), (5,1), (5,2), (5,3), (5,4), (5,6)}

B = {(5,5), (6,4), (4,6) , (6,5), (5,6), (6,6)}

$A\cap B$ = {5,6,), (6,5), (5,6)}

P (B/A) = $\frac{3}{11}$

102. (B)

103. (D)

Let No. of Man = M

Let No. of Women = W

26 M + 21 W = 25 (M + W)

M = 4 W

M : W = 4 : 1

104. (C)

Given $\sin\beta=\frac{2\sin\alpha\cos\alpha}{\sin\alpha+\cos\alpha}$

and $2\sin\theta=\sin\alpha+\cos\alpha$

Statement 1 : $\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)\cdot\sin\beta$

$$=(\sin\alpha+\cos\alpha)\cdot\frac{2\sin\alpha\cos\alpha}{\sin\alpha+\cos\alpha}=2\sin\alpha\cos\alpha=\sin 2\alpha$$

Statement 2 : $\cos\left(\alpha - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$

$$= \frac{2\sin\theta}{\sqrt{2}} = \sqrt{2}\sin\theta$$

105. (B)

$P(A) + P(B) + P(C) = 1$

$\Rightarrow P(A) + \frac{3}{2}P(A) + \frac{1}{2} \times \frac{3}{2}P(A) = 1$

$\Rightarrow \frac{13}{4}P(A) = 1 \Rightarrow P(A) = \frac{4}{13}$

106. (A)

Required probability

$= \frac{25 \times 2}{25 \times 2 + 35 \times 4 + 40 \times 5} = \frac{5}{39}$

107. (C)

Required probability

$= \left({}^8C_6 + {}^8C_7 + {}^8C_8\right)\left(\frac{1}{2}\right)^8$

$= (28+8+1) \times \frac{1}{256} = \frac{37}{256}$

108. (A)

Required Probability

$= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}$

$= \frac{26}{64} = \frac{13}{32}$

109. (C)

By Definition.

110. (B)

Variance will not change by adding or subtracting a fixed value to all the elements.

111. (A)

$(x_1 \cdot x_2 \,........\, x_n)^{1/n} = P$

$(y_1 \cdot y_2 \,........\, y_n)^{1/n} = Q$

$\left(\frac{x_1 \cdot x_2 \,........\, x_n}{y_1 \cdot y_2 \,........\, y_n}\right)^{1/n} = \frac{(x_1 \cdot x_2 \,........\, x_n)^{1/n}}{(y_1 \cdot y_2 \,........\, y_n)^{1/n}} = \frac{P}{Q}$

112. (C)

$P(A) + P(B) - 2P(A \cap B) = q$

$P(A \cap B) = p$

$P(A) + P(B) = 2p + q$

$1 - P(\bar{A}) + 1 - P(\bar{B}) = p + q$

$P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$= 1 - (q + p) = 1 - p - q$

113. (B)

byx = –6, $r = -\frac{1}{2}$

$\left(-\frac{1}{2}\right)^2 = -6 \times bxy \Rightarrow bxy = -\frac{1}{24}$

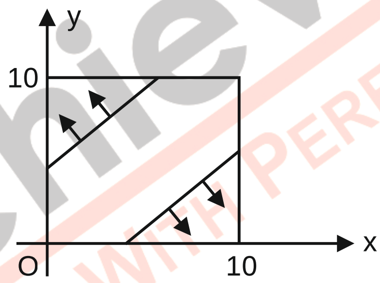
114. (D)

By definition.

115. (C)

Probability $= \frac{2 \times \frac{1}{2} \times 5 \times 6}{11 \times 11}$

$= \frac{30}{121}$



116. (D)

Combined Average

$= \frac{500 \times 1860 + 600 \times 1750}{1100} = 1800$

Combined Variance

$= \frac{500(81 + 3600) + 600(100 + 2500)}{1100}$

$= \frac{(5 \times 3681) + (6 \times 2600)}{1100} \approx 3092$

117. (D)

No. of possible out comes

= 10 + 6 + 3 + 1 = 20

118. (B)

Median can be obtained from ogive.

119. (D)

x = M.D., y = S. D

M.D = $\frac{4}{5}$ S D $\Rightarrow x < y$

120. (C)

By defination.